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# THE PERFORMANCE OF A SPECIAL REPULSION MOTOR IN A CLOSED-LOOP POSITIONING SYSTEM

A THESIS

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THE PERFORMANCE OF A SPECIAL REPULSION MOTOR

IN A CLOSED-LOOP POSITIONING SYSTEM

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To my wife, I express appreciation for her love, patience and understanding and also for her assistance and encouragement in the preparation of the final draft.

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# SUMMARY

In this study, a positioning servomechanism with high torque output was designed and tested. The high torque output was provided by a special repulsion motor. A novel feature is used in the special repulsion motor to provide the reversing action required by the positioning system. In order to reverse the motor, the necessary shift of the relative position of the brush axis with respect to that of the stator field was accomplished by shifting the stator field electrically, with the brush axis remaining in a fixed position.

This feature was provided in the special repulsion motor by using a lap winding on the stator which had a set of power input taps each side of a reference axis that coincides with the brush axis. By switching the a-c power input from one set of taps to the other set, a shift in the relative position of the stator field with respect to the brush axis is obtained, which reverses the direction of rotation of the motor.

The proper magnitude of the output torque was obtained by controlling the a-c power input to the motor. The primary of a transformer was connected between the a-c line and one set of the power input taps on the motor (another transformer being used for the other set of taps). The effective impedance between the a-c line and one set of motor taps could then be decreased by short circuiting the secondary of the transformer with a pair of thyratrons. By varying

the ignition angle of the thyratrons the power input to the motor could be controlled. The direction of rotation was determined by which of the two transformers and thyratron combinations were operated.

Stalled torque tests and operating performance tests were performed on the motor. The stalled torque of the motor is 4.4 lb-ft with 50 volts input. At 50 volts input, the best efficiency of 47 per cent occurs at approximately 1500 rpm with an output of 324 watts.

An open-loop frequency response test, a closed-loop frequency response test and a velocity test were performed on the complete system. The system operated satisfactorily as a positioning servomechanism. The gain of the system, the dead zones, friction, etc. were such as to make the position error negligible. The maximum velocity error was 3 degrees. The frequency response of the system was flat to approximately one cps.

# CHAPTER I

## INTRODUCTION

Closed-loop positioning systems are widely used for military and industrial purposes. In some applications, in order that the output load will accurately follow the input signal, high torque levels are required. The high torque levels may be provided in several ways with varying degrees of complexity. Direct current motors are used in this type of application as well as hydraulic motors and actuators. Another such device which will provide high torque levels is the repulsion motor. It also has the particular advantage of operating directly from available a-c power. Theoretically, the starting torque per ampere is higher for the repulsion motor than for any other, being 5 to 7 oz-ft per ampere at 110 volts (1).

While the repulsion motor possesses some particular advantages, there are problems involved in utilizing it in a closed-loop system. The powering device of a positioning system in addition to being able to provide the necessary magnitude of output torque, must have a means of reversing the direction of the output torque. While several schemes have been used for reversing the direction of rotation of the repulsion motor, the most common method accomplishes this by mechanically shifting the position of the brushes. In a repulsion motor, the relative position of the axis of the brushes with respect to the axis of the field windings determines the direction as well as the magnitude of the output torque.

In general, mechanically shifting the position of the brushes would be an unsatisfactory method of controlling the output torque in a positioning system.

One method which eliminates the difficulties in mechanically shifting the brushes was evaluated by T. E. Flanders (2). In this method, to provide the reversing action, two brush sets are used. They are arranged in such a manner that the use of one or the other of the brush sets provides the proper direction of rotation. The selection of the proper brush set is determined by an electrical control circuit that responds to the error signal of the positioning servomechanism.

For the system of this study, the magnitude and direction of the output torque of the repulsion motor is controlled by electrically shifting the axis of the field winding with respect to the brush axis which remains in a fixed position. As the stator windings conventionally used in repulsion motors were unsatisfactory for this, the stator of the repulsion motor used in this study was provided with a special winding.

In this study, a motor was built and the characteristics of the motor were determined. Also suitable circuitry for controlling the motor was developed, and the performance of the positioning system was evaluated.

# CHAPTER II

## THE SYSTEM AND ITS OPERATION

The Basic System.--The block diagram of the basic positioning system is of the form given in Figure 1.

The block diagram given in Figure 1 can be expanded into a more detailed block diagram of Figure 2. Rate feedback as well as position feedback is used. Therefore the system may be classified as a proportional-error servomechanism with first-derivative output control (3).

The basic operation of the control system is as follows: Assume an input signal is applied to the system by the rotation of the synchro control transformer shaft. The difference in shaft positions of the synchro control transformer and the synchro generator, or the error signal, is applied to the a-c amplifier. The synchro control transformer and synchro generator operate from a 60-cycle power source. The error signal input to the amplifier has a suppressed carrier modulation. The output of the a-c amplifier is demodulated. The difference between the d-c level of the demodulated signal and the d-c tachometer feedback is applied to the motor control circuits. The motor control circuits apply a-c power to the repulsion motor to produce, $\Theta_o$, which is the output rotation of the motor shaft. The motor shaft drives the shaft of the synchro generator through a gear train until the position of the synchro generator corresponds with that of the synchro control transformer.

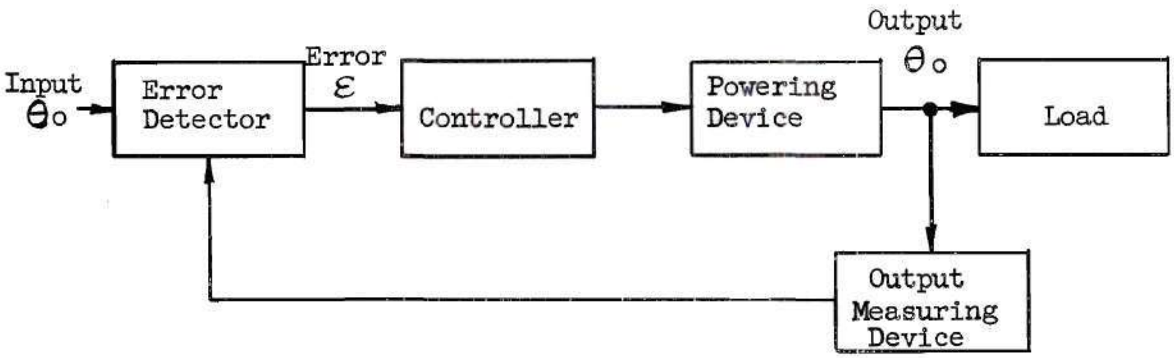



Figure 1. Block Diagram of the Basic Positioning System

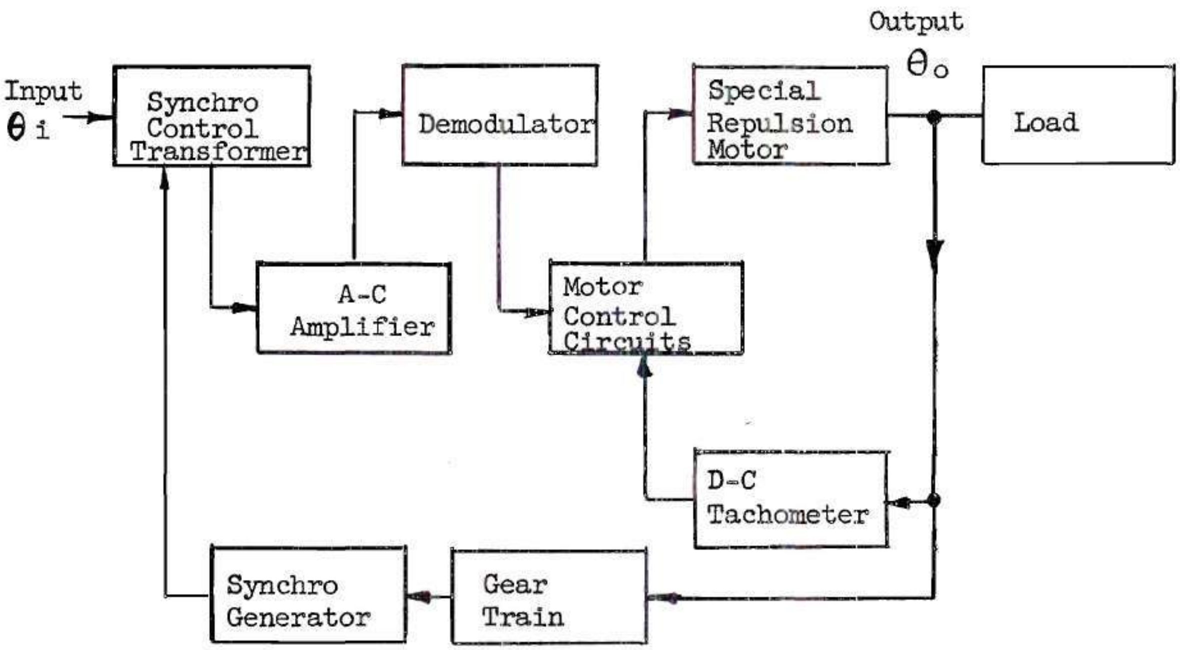



Figure 2. Block Diagram of the Experimental Positioning System

Component Parts of the System.--The synchro control transformer and the synchro generator form the error detector and the output measuring device (see Figure 3).

The a-c amplifier is used to amplify the error signal (see Figure 4). The amplifier has a gain adjustment so that the sensitivity of the system may be adjusted for best performance.

The demodulator is a conventional sum and difference full-wave demodulator (see Figure 5) (4).

The powering device of the control system is a repulsion motor with a modified stator winding. The stator is provided with a lap winding. With respect to an electrical neutral, taps are brought off the winding 20 electrical degrees each side of the electrical neutral (see Figure 6). The axis of the brushes of the repulsion motor are set in line with the electrical neutral. Therefore, if the switch is closed in position AA, the relative position of the stator field is displaced 20 degrees clockwise from that of the brush axis and the motor will run clockwise. If the switch is closed in position BB, the relative position of the stator field is displaced 20 degrees counter-clockwise from that of the brush axis and the motor will run counter-clockwise. The operation of this motor is then similar to that of a conventional repulsion motor whose brushes are mechanically shifted 20 degrees either side of the electrical neutral.

In the control system, an electrical circuit replaces the double pole double throw switch (see Figure 7). The primaries of the transformer are in series with the motor and the a-c line. The primaries

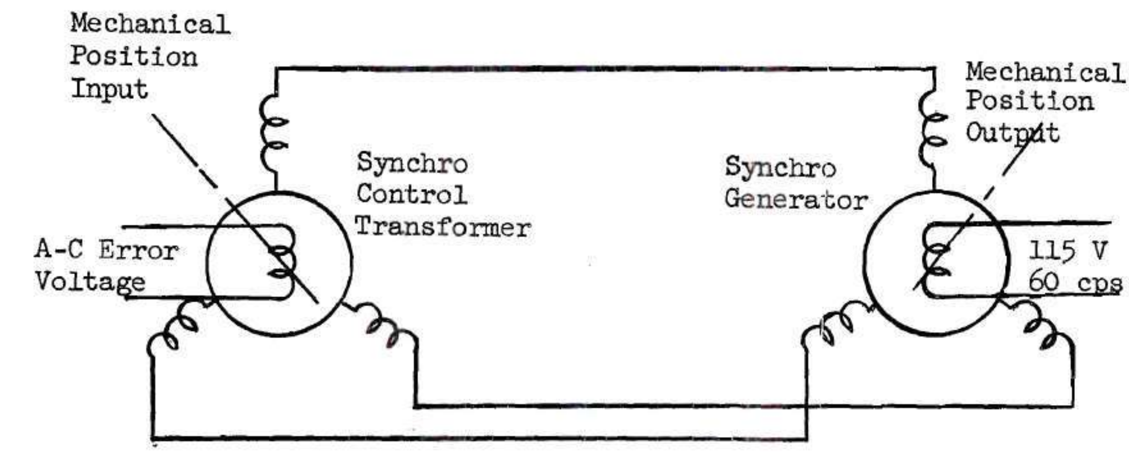



Figure 3. Position Error Detector and Output Measuring Device

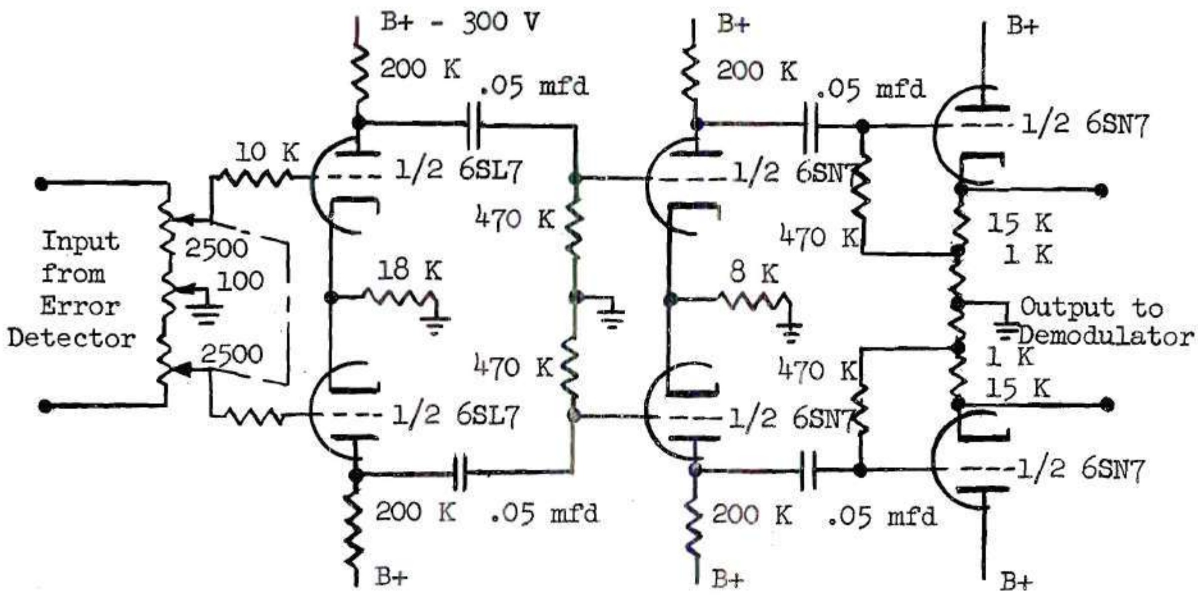



Figure 4. A-C Amplifier

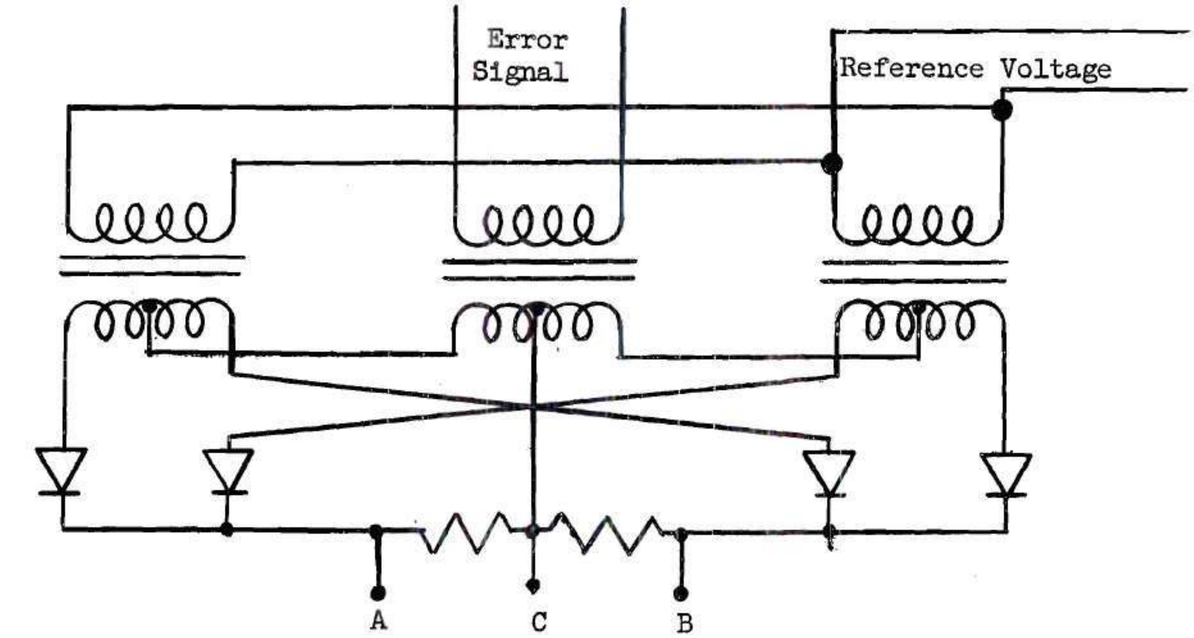



Figure 5. Sum and Difference Full-Wave Demodulator

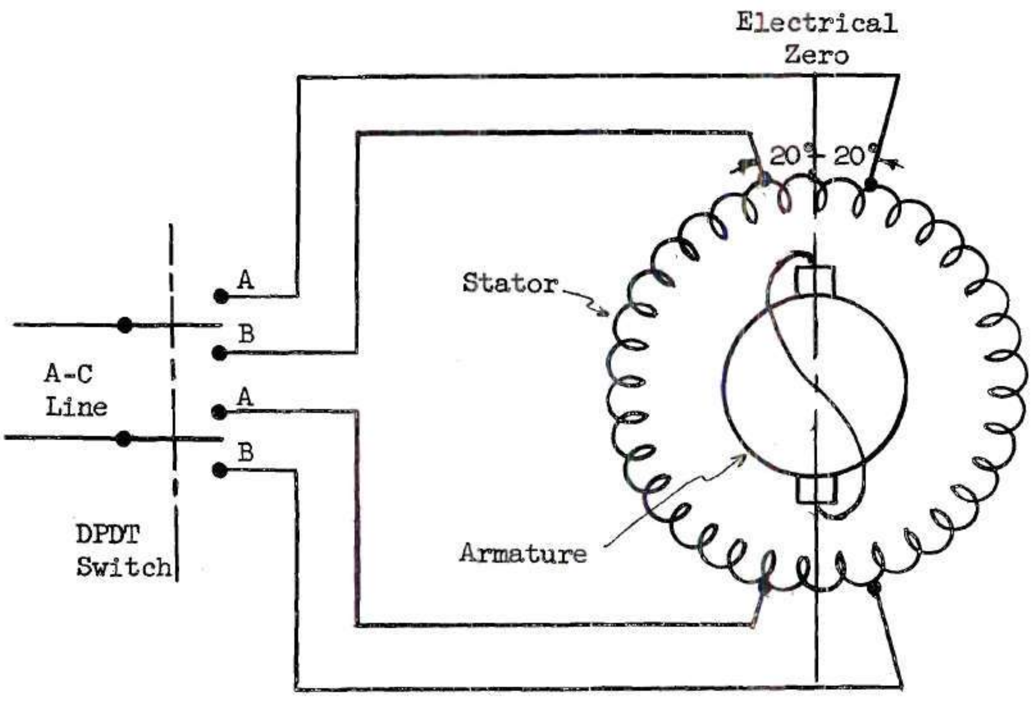



Figure 6. Basic Reversing Circuit for the Special Repulsion Motor

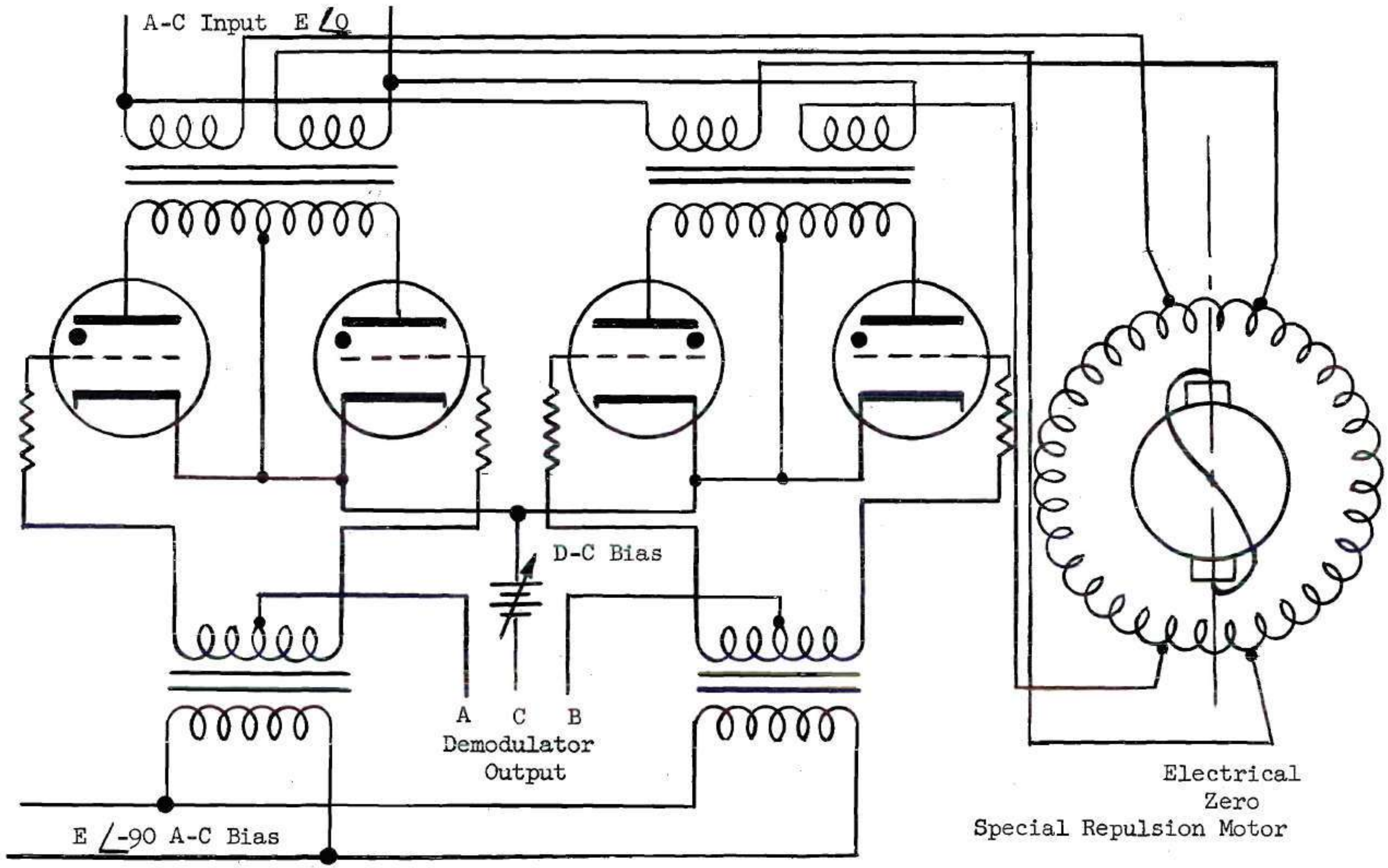



Figure 7. Basic Motor Control Circuit

of the transformer are in series with the motor and the a-c line. The primaries of each transformer consist of two windings in order to provide electrical isolation between adjacent taps on the motor. Across the secondary of each transformer are a pair of thyratrons. When one pair of thyratrons is conducting, the secondary is short-circuited and the primary of this transformer offers little impedance between the a-c line and the motor. The direction of rotation of the motor is determined by which combination of transformer and thyratrons are operating. Under this condition the other transformer and thyratrons are not operating.

Under quiescent conditions, both pairs of thyratrons are cut-off. The grids of the thyratrons are biased with a conventional arrangement of d-c and a-c voltages which can be referred to as bias shift control (5). The a-c grid bias voltage is shifted 90 degrees with respect to the anode voltage and is on the order of 100 volts peak-to-peak. Sufficient d-c bias is applied to keep the thyratrons cut-off under quiescent conditions.

This bias arrangement allows continuous control of the thyratron firing angle from zero to 180 degrees of a cycle thus providing smooth control of the motor output. The firing angle of a particular set of thyratrons is of course determined by the relative amplitude of the input signal which is a positive d-c voltage at either point A or B with respect to point C (see Figure 7).

The tachometer has a d-c output voltage whose polarity depends upon the direction of rotation and has an output of approximately

2 volts per 100 rpm. The tachometer is geared to the motor shaft in a one to one ratio and is connected to the motor control circuit as shown in Figure 8.

As is apparent from Figure 8, the tachometer voltage would normally appear across the two resistors which would cut the value of the feedback voltage to the operating thyratrons in half. This difficulty is overcome by using diodes to short circuit the resistor of the non-conducting pair of thyratrons and thus allowing the full feedback voltage to appear in series with the pair of operating thyratrons.

The load for the control system consists of an inertia and coulomb friction. Most of this inertia is that of the motor armature and the coulomb friction is due to the brushes of the motor. A flywheel was used to provide additional inertia in determining the load parameters (see Appendix A).

Complete System.--The complete control system which is composed of all the preceding components is shown in Figure 8. The operation of the control system is essentially that previously given for the block diagram. The manner in which the system reverses direction is explained in the explanation of the individual components.

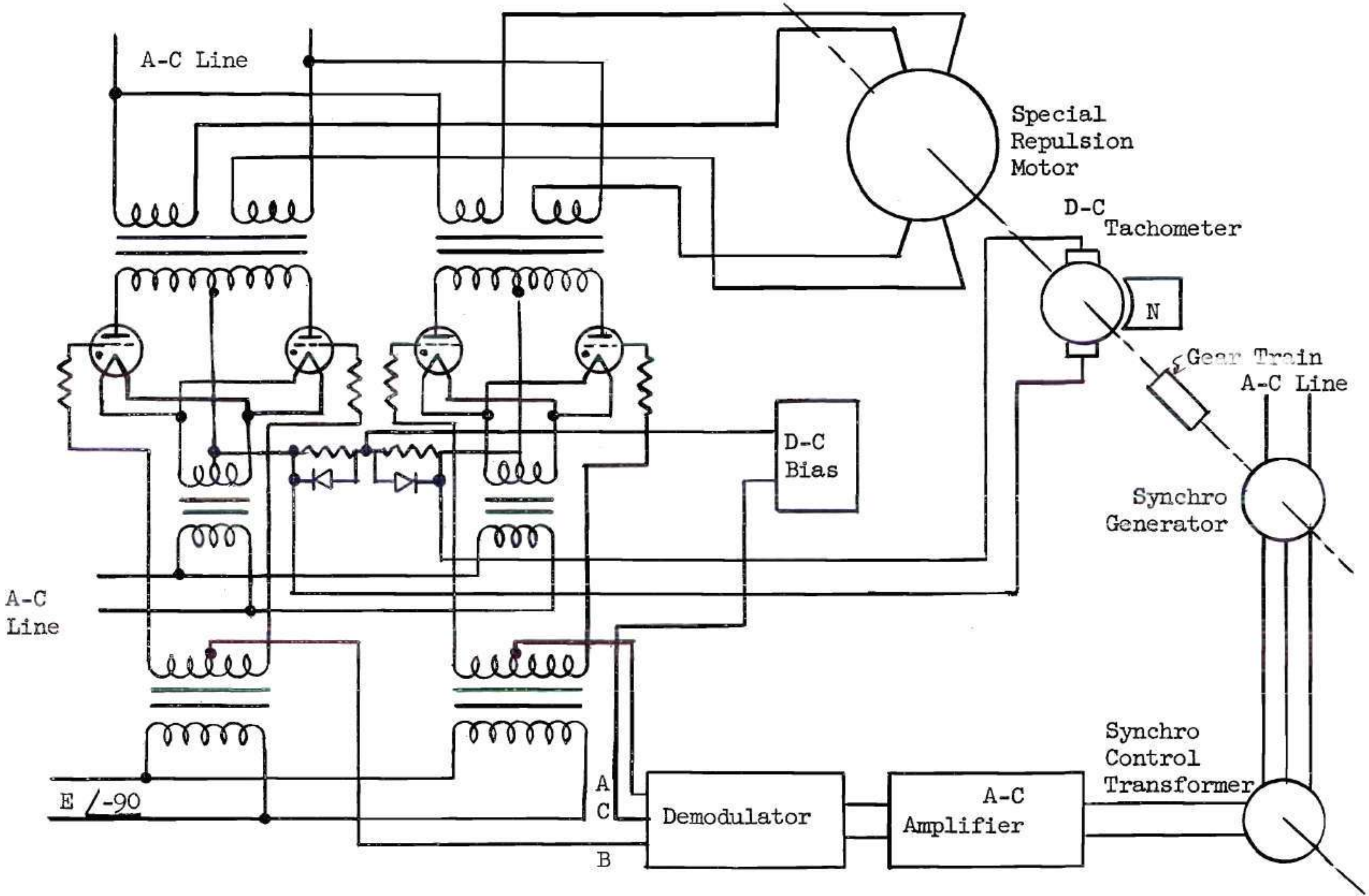



Figure 8. Diagram of the Complete Positioning Servomechanism

# CHAPTER III

## DESIGN AND CONSTRUCTION OF EQUIPMENT

Construction of the Motor.--As a motor of the proper construction was not available, it was necessary to build one (see Figure 9). The armature used in the repulsion motor was obtained from a four-pole d-c motor. The stator and frame were part of a one-third horsepower induction motor. It was necessary that the armature be turned down slightly in order to obtain proper clearance with the stator. The magnetic paths of the armature and stator were two inches wide. No further modification of the armature was required. It was necessary to completely rewind the stator. The stator winding used was a double layer, lap, full pitch winding. As there were 36 slots in the stator, 36 coils were required. In order to obtain maximum utilization of slot space, 20 turns of number 18 Heavy Formvar wire were used in each coil. The dimensions of the coils proved quite critical and required special forming.

The optimum operating voltage of 50 volts was determined experimentally (see Chapter IV, "Performance of the Special Repulsion Motor").

The stator has 36 coils so each coil was 10 mechanical degrees apart. However as this is a four-pole machine, each coil was 20 electrical degrees apart. With respect to an arbitrary reference or zero degree coil, four sets of taps were brought out from the lap winding on the stator (see Figure 10). Each set of taps consisted of three wires.

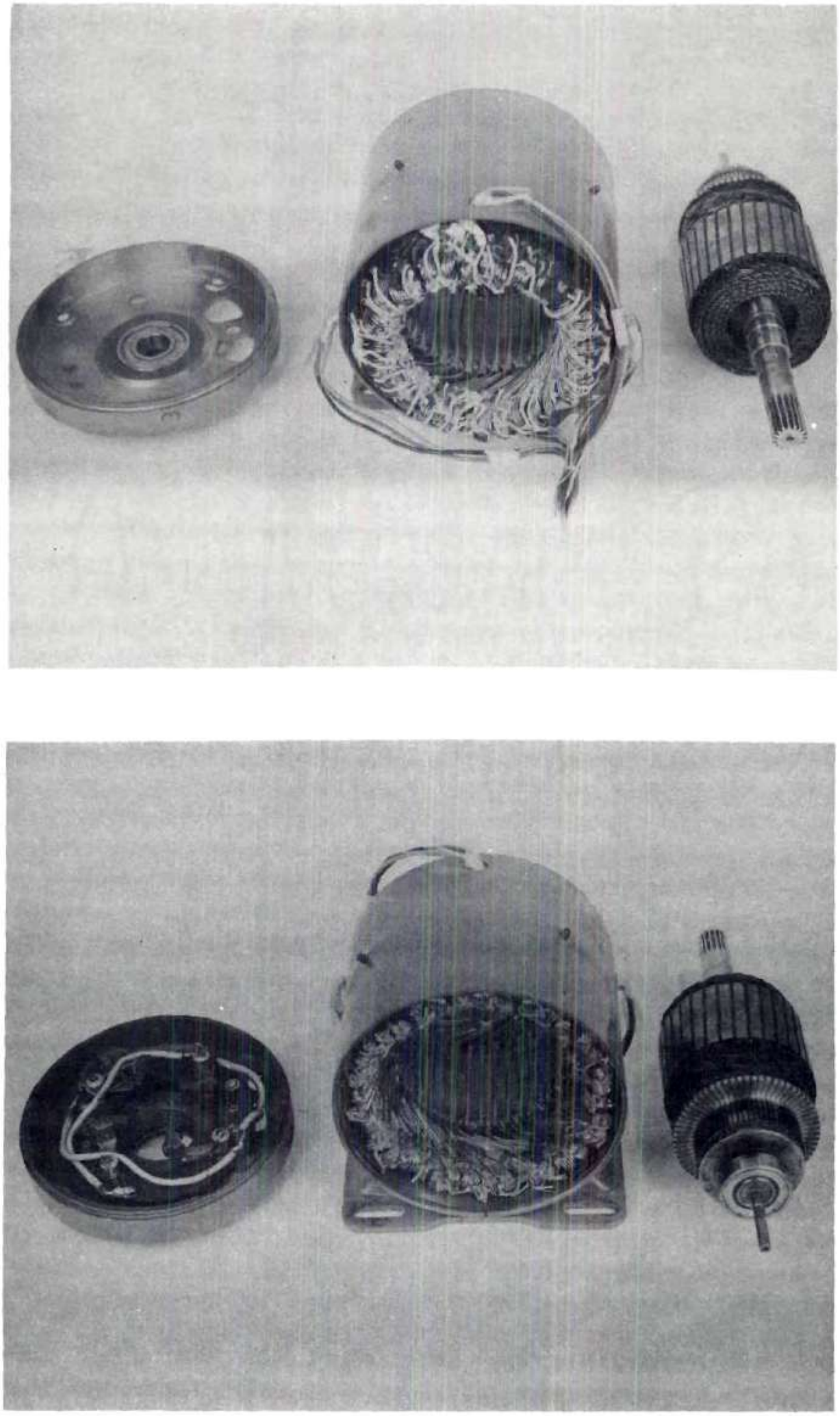
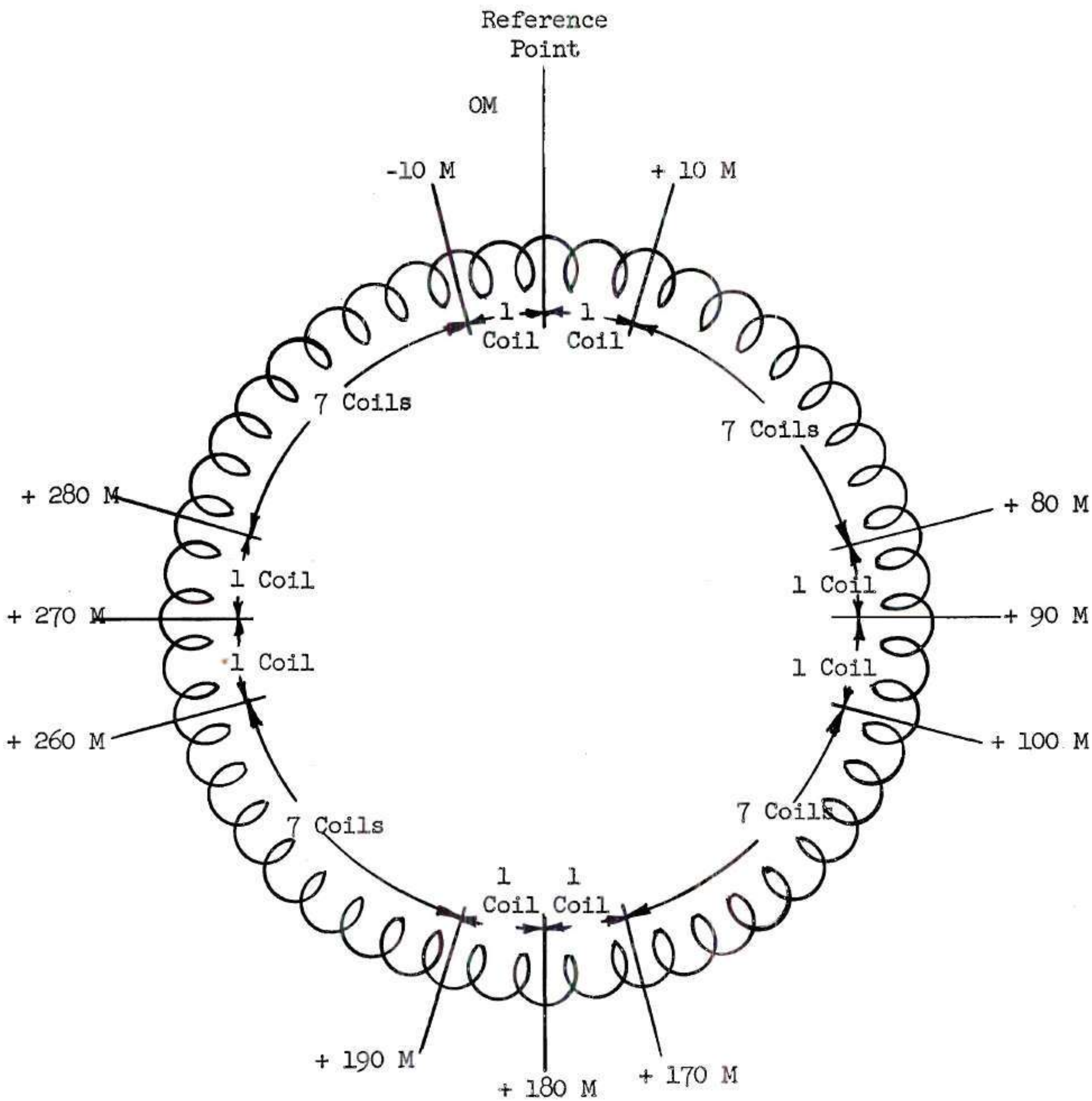


Figure 9. Photographs of the Special Repulsion Motor




M = Mechanical Degrees
10 Mechanical Degrees Equal 20 Electrical Degrees

Figure 10. Location of the Taps on the Stator Winding of the Four Pole Motor

With respect to the zero degree coil, the relative position of the taps are located as shown in Figure 10. Later tests proved that the electrical balance of the stator winding was good and therefore electrically equivalent taps could be connected together without causing adverse effects (see Figure 11). The balance taps were used only during construction to help determine the proper location of the brush axis. This was useful in determining the approximate location; the exact brush axis was determined using stalled torque tests (see Chapter IV, "Performance of the Special Repulsion Motor"). With the brush axis properly positioned, if a-c power is applied to the counter-clockwise taps, the motor will rotate in a counter-clockwise direction. If a-c power is applied to the clockwise taps, the motor will rotate in a clockwise direction.

Motor Control Circuit.--In order to utilize available transformers, it was necessary to modify the motor control circuit from that given in Figures 7 and 8. The modification is slight and in general operates in the same manner as previously described for the simpler circuit (see Figure 12).

Transformer T-1 was used in order to obtain the proper primary voltage for T-2. Also a choice of several taps was available on the secondary of T-1 and the primary of T-2 which allowed the adjustment of the output voltage. Secondary winding S-1 and S-2 of transformer T-2 were in series with the primary of T-3 also secondary winding S-3 and S-4 of transformer T-2 were in series with the primary of T-4. Transformers T-3 and T-4 had step-up ratios of one to ten. All transformers

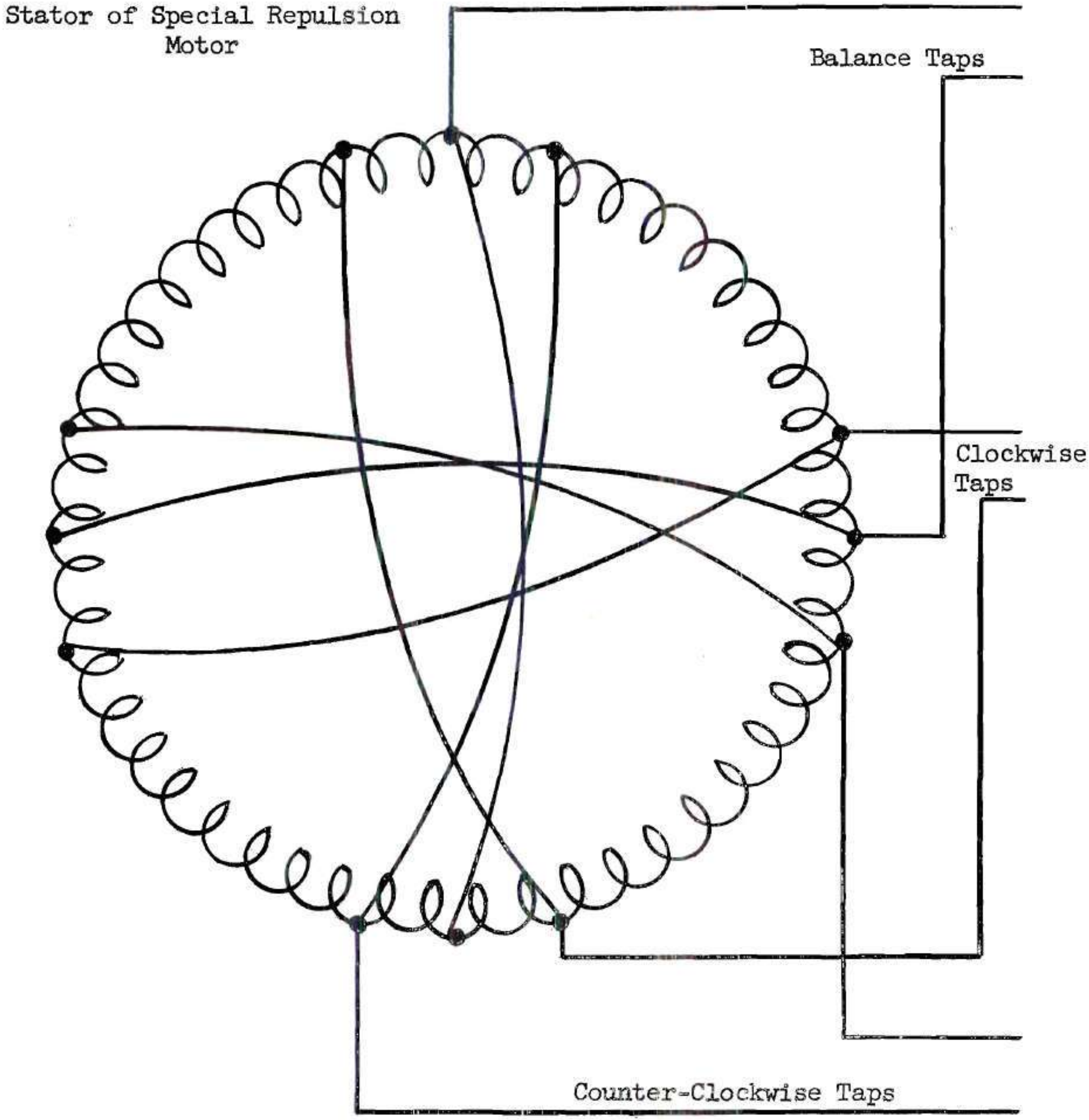



Figure 11. Interconnection of Electrically Equivalent Taps

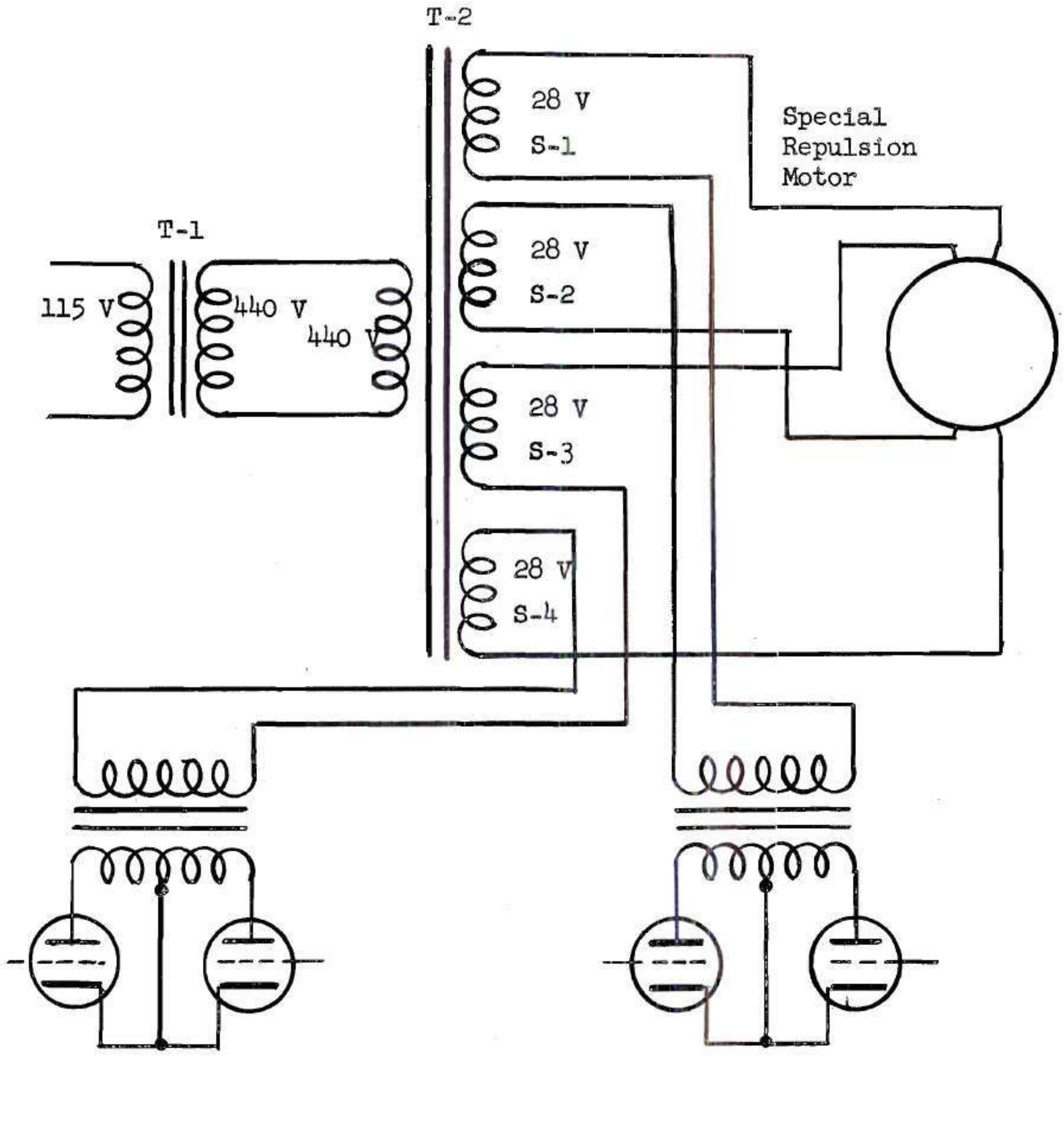



Figure 12. Modified Motor Control Circuit

had adequate power handling capabilities and were used within the voltage ratings.

The main consideration then became that of evaluating the maximum demand imposed on the thyratrons. The voltage across the transformer T-3 and T-4 occur when the thyratrons are cut-off. The maximum primary voltage was approximately 60 volts rms. For the secondary, the maximum peak inverse voltage was then

$$(\sqrt{2})60 \times 10 = 846$$

The thyratron used was the FG-17 which has a maximum peak inverse anode voltage rating of 2500 volts. Therefore maximum voltage presented no problem. From the motor performance tests, (see Chapter IV, "Performance of the Special Repulsion Motor"), the motor current was found to have a peak value of approximately 42 amperes under stalled conditions.

The peak thyratron current under this condition has an instantaneous value of approximately 4.2 amperes. This exceeds the rating of the thyratrons which is given as two amperes for instantaneous maximum current for 25 cycles and above. In normal operation, this value of current is obtained at most for only one or two cycles during the starting transient. After the starting transient, the motor current decreased to approximately 13.2 amperes rms (for the normal loading imposed by the servo system). This would give an average value of approximately 0.4 ampere. The rated maximum average anode current is given as 0.5 amperes when averaged over a maximum time of 15 seconds. The values of current given above were obtained when the grid bias of the thyratrons was such as to allow the thyratrons to operate for a full half cycle. Even though the thyratrons were operating slightly outside the rating given by the manufacturer, no particular difficulty was encountered.

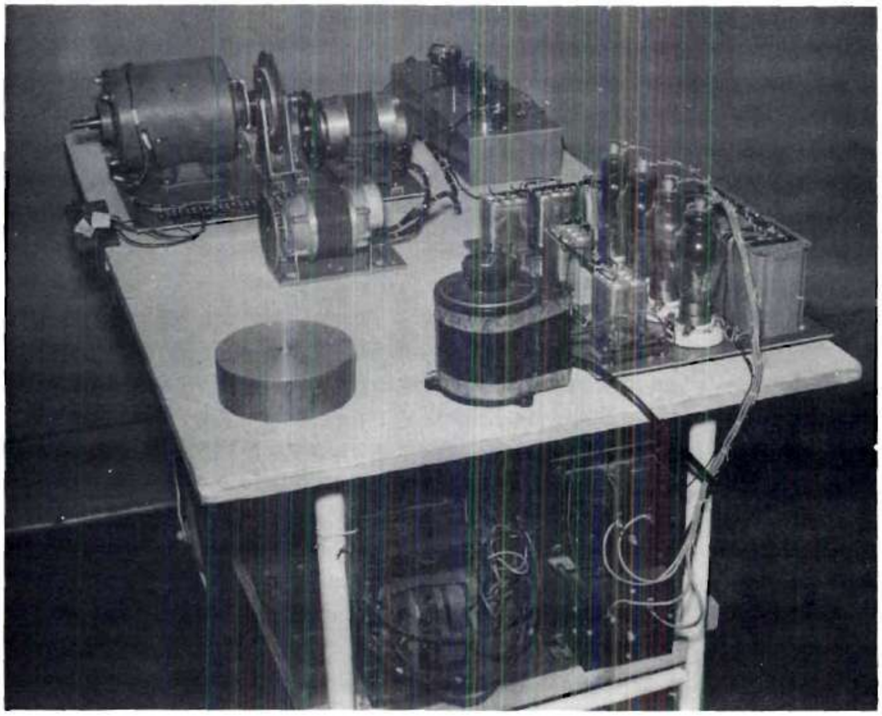
Error Detector.--The synchro generator was geared to the motor through a gear train with a step down ratio of 43.2 to one. An antiback lash gear was used on the synchro generator shaft. This helped to reduce the problems resulting from back lash. The synchro generator, gear train and motor were mounted on a piece of 0.125 aluminum sheet which had been stiffened with aluminum channels. The synchro generator and gear train were mounted in such a manner as to allow the step down ratio to be variable over a range of from 25 to one down to 70 to one. The maximum output of the synchro control transformer was approximately 54 volts rms.

Tachometer.--The d-c tachometer was coupled to the motor shaft through a one to one gear ratio. This ratio being chosen in order to allow the maximum output voltage and still not damage the d-c tachometer. Later tests indicated the output was linear with rpm and was 19.5 volts per 1000 rpm.

Load.--The load for the control system consists of an inertia and the damping or friction torques. For this study the inertia and friction torques comprising the load are those of the motor, gear train and synchro generator. A test was run to determine their values. This test including the result is discussed in Appendix A. The load inertia is essentially that of the motor which is 0.002 slug-ft$^2$. The friction torque is essentially the coulomb friction of the motor which is 0.068 lb-ft.

Amplifier.--The amplifier was an r-c coupled push-pull amplifier with cathode follower output (see Figure 4). An output of approximately 75 volts rms could be obtained without distortion. The amplifier gain during the tests was set at 17 and 59. The large output of the synchro control transformer could easily saturate the amplifier. This initially caused some difficulties with proper demodulator operation until a capacitor of 0.1 mfd. was inserted across the primary of the input transformer to the demodulator. The optimum value of this capacitor was determined from tests with the amplifier and demodulator.

System Diagram.--A photograph of the complete system is shown in Figure 13.



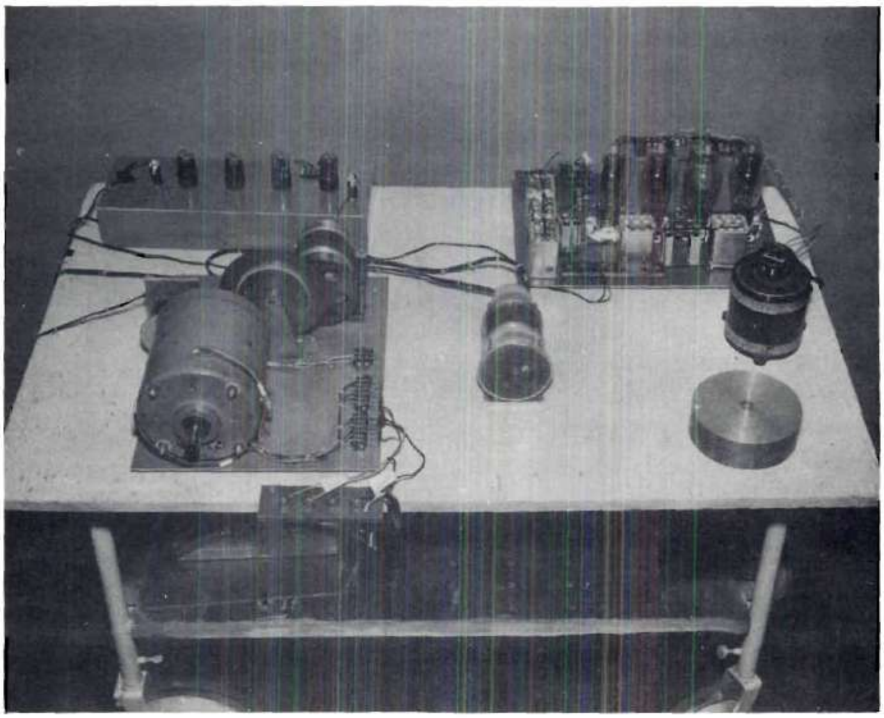


Figure 13. Photographs of the Positioning Servomechanism

# CHAPTER IV

## PERFORMANCE OF THE SPECIAL REPULSION MOTOR

After the motor was constructed, tests were performed in order to determine the operating characteristics. These tests include stalled torque tests, load tests, and brush position tests. The dynamometer test stand, General Electric Model 9742799G2, was used in the stalled torque tests and load tests.

A preliminary test was performed in order to determine the maximum voltage which could be applied to the stator winding of the motor before saturation of the iron occurred. This was performed by connecting an autotransformer to one set of taps on the stator windings, i.e., either the clockwise taps or the counter-clockwise taps. The short circuit across the brushes was removed and the brushes were connected to the Y axis of an oscilloscope so the voltage wave-form in the armature could be observed. The voltage input to the stator winding was increased until the sinusoidal wave form of the armature voltage became distorted. This occurred at approximately 60 volts.

A test was then conducted to determine the proper brush position. It was necessary to align the brush axis with the zero electrical degree axis of the stator. In the first method tried, the input voltage was connected to the balance taps. The brushes were short circuited as normal and the armature of the motor was blocked to prevent rotation.

The brush position was then moved until maximum input current was obtained. Due to large variations in input current for small motions of the motor armature, only the approximate location of the brushes was determined in this manner.

The proper location for the brushes was more accurately determined by observing the relative output stall torque for various brush positions. Because of the mechanical construction of the motor, the relative position of the input taps on the stator winding is fixed plus or minus 20 electrical degrees on each side of the electrical zero. Therefore in order to obtain equal torque output in both directions of rotation, the brush axis must coincide with that of the electrical zero. The maximum torque output in a given direction does not occur however when the relative position of the taps to the brush axis is 20 degrees.

A test was run to determine the proper position of the brushes for maximum stalled torque in a given direction of rotation. The position of the brushes for maximum stalled torque in the opposite direction of rotation was then determined. The brush holder had been provided with a degree scale and a pointer was attached to the motor frame, so the relative degree readings of each maximum torque points were observed. It then became a simple matter to set the brushes at the mid-point of the two readings. The test was repeated several times with different armature positions and the average reading was taken. The test was simplified somewhat due to the maximum stalled torque point being very close to the 20 electrical degree setting. The maximum stalled torque point occurred at approximately 18 electrical degrees. The maximum stalled torque is about 10 per cent greater than the stalled torque at the 20 degree setting. Therefore for all practical purposes the 20 degree setting imposed by mechanical limitations is an optimum setting.

The stalled torque tests and the load tests were performed on the General Electric Model 9742799G2 dynamometer test stand. For the stalled torque tests the armature and the stator of the d-c generator were locked together. Stalled torque tests were performed with various input voltages to the stator and the data was recorded as given in Appendix B. A graph of output torque and stator current versus stator voltage is shown in Figure 14.

The torque exerted by the armature of a motor is proportional to the air gap flux and the armature current. The armature is inductively coupled to the stator. Therefore as the stator current is increased, both the motor armature current and field current is increased. The torque in such a case would then be proportional to the square of the stator current. This however is not the case and comparison of the data at 30 volts and 50 volts input to the stator indicates that when the stator current is doubled the output torque increases to slightly over 2-1/2 times the initial value.

For purposes of comparison, another repulsion motor rated at 1/3 hp has a stalled torque of 4 lb-ft. This special repulsion motor at the selected operating voltage of 50 volts has a stalled torque of 4.4 lb-ft.

The load tests were run with various input voltages to the stator and the data was recorded as given in Appendix C. The motor

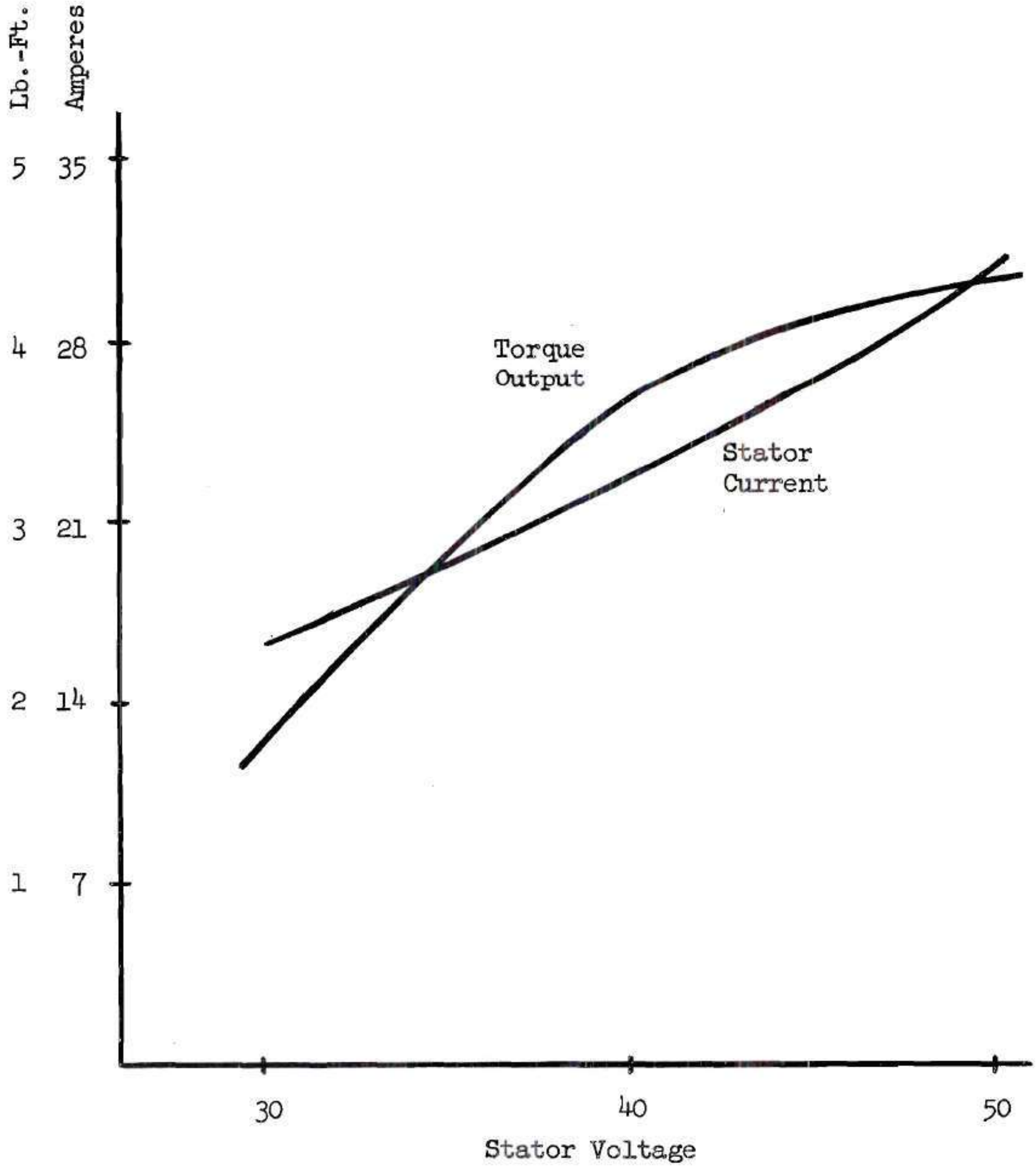



Figure 14. Stalled Torque Test of the Special Repulsion Motor

was not loaded as heavily at 55 and 60 volts due to the overheating of the motor. Sufficient data were taken to show the general performance however. The graph at 50 volts input is shown in Figure 15 (6).

For 50 volts input to the stator winding, the maximum output of slightly less than 1/2 hp occurred at approximately 1100 rpm. Of interest is the fact that the maximum power output was developed at approximately this rpm with 40 volts and 45 volts applied to the stator winding. The tests,run with 55 and 60 volts,were not taken to rpm values as low as this so no comparison can be made at these voltages. The best efficiency for 50 volts input which is 47 per cent occurs at about 1500 rpm. The output power is approximately 324 watts. This compares slightly better than for 45 volts input which has an efficiency of 46 per cent with an output of 247 watts which occurs at about 1400 rpm. At 55 volts input, with 46 per cent efficiency, a power output of 370 watts is obtained at about 1500 rpm. At 40 volts and 60 volts the best efficiency is several per cent less. From the standpoint of efficiency, an input voltage of 50 or 55 volts appears to be the best. More torque and output power is available at the higher input voltage, however, the lower voltage is probably the better selection due to the tendency of the motor to overheat. The no load speed of the motor with 40 volts input was 3520 rpm and with 50 volts the input was 3820 rpm.

For all practical purposes motor performance is the same in both directions of rotation. The test results for 45 volts input in both directions of rotation are given in Tables 5 and 9, Appendix C. This also reaffirms that the brushes were properly positioned.

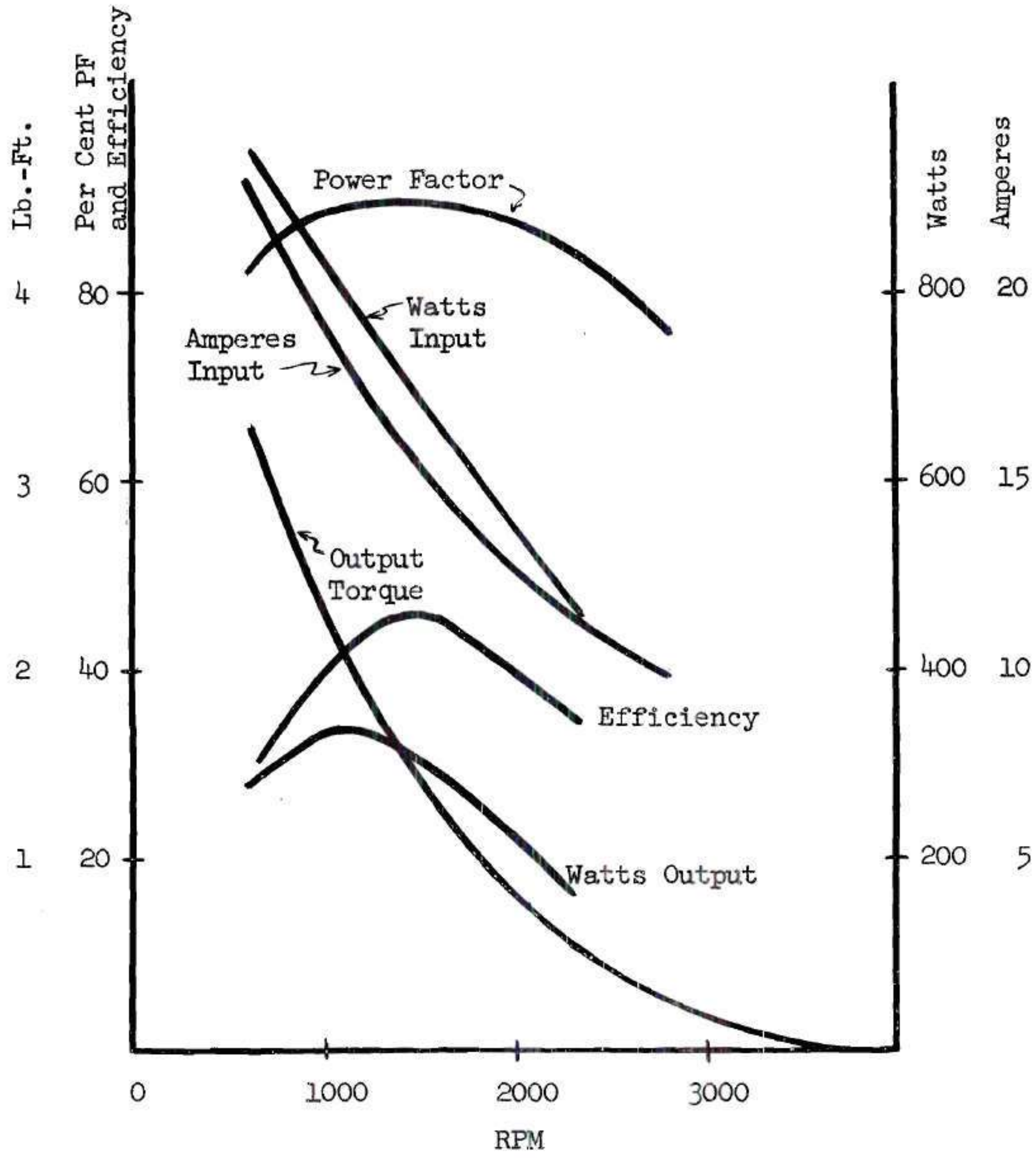



Figure 15. Load Test of the Special Repulsion Motor with 50 Volts Applied to the Stator

# CHAPTER V

## PERFORMANCE OF THE FINAL SYSTEM

In order to evaluate the performance of the control system a steady state test and a dynamic response test were used (7). The possible steady state tests include the constant position, the constant velocity, and the constant acceleration test. The possible dynamic tests include the transient response test and the frequency response test.

The steady state test was used to determine the response of the control system to a steady state input. For this control system the input signal is applied to the shaft of the synchro control transformer and the output is monitored by the synchro generator. Under quiescent conditions, the relative position error between the synchro control transformer shaft and the shaft of the synchro generator is zero degrees. When an input signal is applied, depending upon the type, an error or difference in the positions of the two synchro shafts may result. The steady state test was performed in order to determine the amount of this error.

For this control system the constant position error is zero, i.e. when the shaft of the synchro control transformer is rotated from the quiescent position to a new position, the output will follow this until the synchro generator shaft is in alignment with that of the synchro control transformer. The gain of the system, the dead zones, friction, etc. were such as to make the position error very small. Due to non-linearity of the synchros, there may have been a position error at particular shaft positions, however this defect of the individual components is small and is not considered in this study.

For this control system, the constant acceleration error is for all practical purposes infinitely large, i.e. when the shaft of the synchro control transformer is rotated with a constant acceleration from the quiescent position, the position error of the output shaft with respect to the input will increase in magnitude until all correspondence between the two is lost.

When a velocity input signal is applied to this control system, a finite velocity error exists. A test was performed in order to determine the magnitude of this error. The test setup is shown in Figure 16. A series a-c motor operating through a speed reducer was used to rotate the shaft of the synchro control transformer. The speed of the series a-c motor was controlled by varying the input voltage with an auto-transformer. The repulsion motor speed was determined from the output of the d-c tachometer. The speed of rotation of the synchros could easily be determined from the motor speed as a 43.2 to one ratio gear train was used between the motor and the synchro generator. The position error between the synchro control transformer and the synchro generator was determined from the magnitude of the error signal. As 60 cycle suppressed carrier modulation is used, a demodulator was required in order that the magnitude of the error signal could be recorded on the Sanborn Recorder. A two channel Sanborn Recorder was used with the

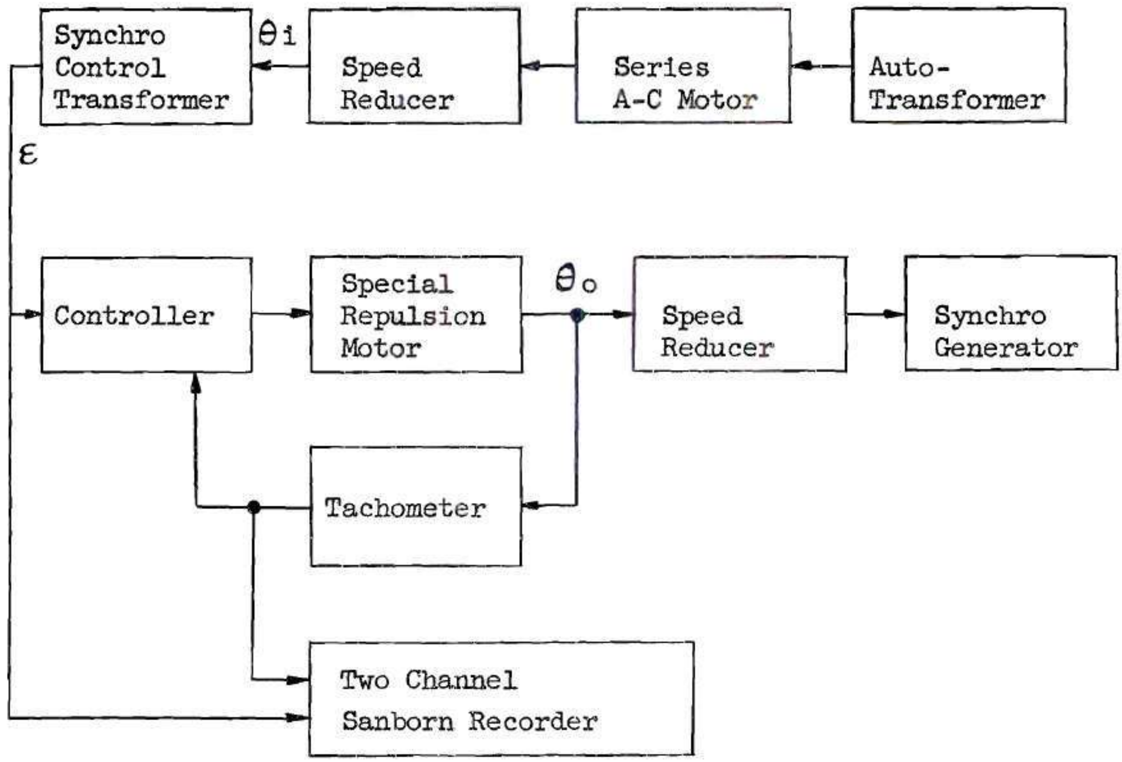



Figure 16. Equipment and Arrangements for Performing Velocity Test

error signal being recorded on one channel and the d-c tachometer output being recorded on the second channel.

The test was performed for various values of input velocity. The data are given in Table 10 of Appendix D. The values given in Table 10 are average values. When the input of the control system was rotated at a constant angular velocity, the output shaft followed the input, but tended to oscillate around a given value of position error without damping out to a steady value. This was caused by backlash in the gear train between the repulsion motor and the synchro generator which was aggravated by the high sensitivity of the system. The results are given in Figure 17. It is apparent from the figure that the maximum error in the position of the shaft of the synchro control transformer and in position of the shaft of the synchro generator is approximately three degrees. Above this value the control system is operating in saturation and the loss of control is quite likely to occur. The velocity of the input to synchro control transformer with the three degree position error is approximately 46 rpm. Examination of the motor performance test data indicates that the maximum velocity could possibly be increased 10 to 20 per cent at the expense of other performance characteristics of the control system. This might be accomplished by removing the rate feedback loop. However as the control system is unstable without this feedback, the gain of the system therefore must be reduced in this case. A second method to accomplish this without producing undesirable changes in system performance, would be to redesign the amplifier so that saturation occurs at a higher value of amplifier output.

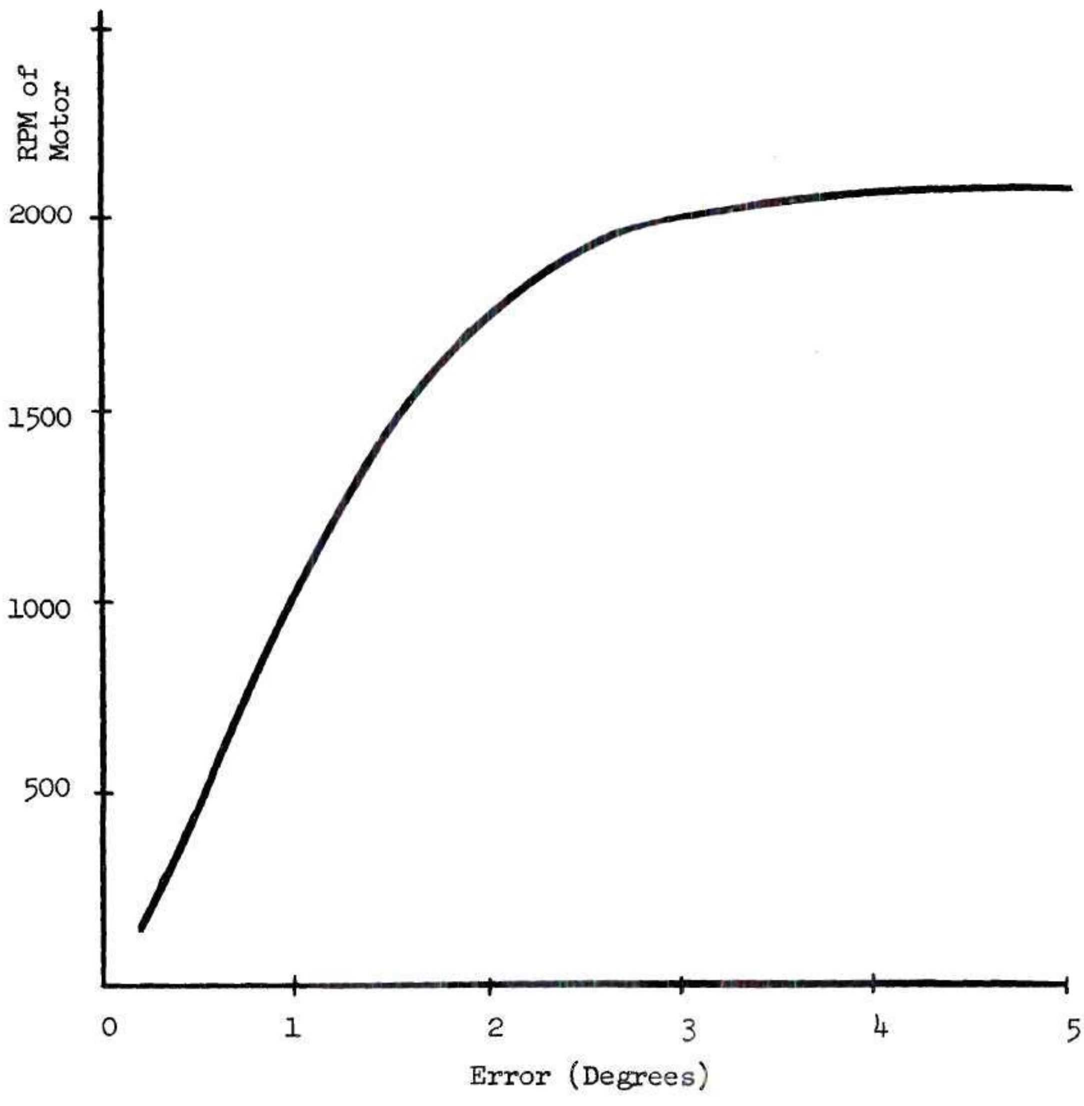



Figure 17. Velocity Test - Motor Speed Versus Degrees Error

Of the two possible types of dynamic tests, the frequency response test was used to evaluate the control system. One transient test was performed in which the closed-loop system was subjected to a step position input signal. However due to the difficulties with the gear train between the repulsion motor and the synchro generator, uniform results could not be obtained, i.e. the transient damped oscillation was greatly affected by relative gear positions.

Two frequency response tests were performed on the control system. In one test the open-loop response of the system was determined directly using the open-loop system without either position or derivative feedback. The second test was used to evaluate the closed-loop response of the system directly using the closed-loop system with both position and derivative feedback.

The test set-up used to determine the open-loop frequency response of the control system is shown in Figure 18. To simplify testing, the Servoscope Model 1100A was used in place of the synchro control transformer and amplifier. The output of the amplifier was removed from the signal input transformer of the demodulator and the output of the Servoscope was substituted. The Servoscope output was set at a level which would simulate low input signal operation. This was desirable as the high gain amplifier will saturate the output of the control system for small displacements of the synchro control transformer shaft. Thus it was desirable to use a low level input signal in order to minimize the distortion of the output signal. In order to eliminate difficulties due to zero position drift in the

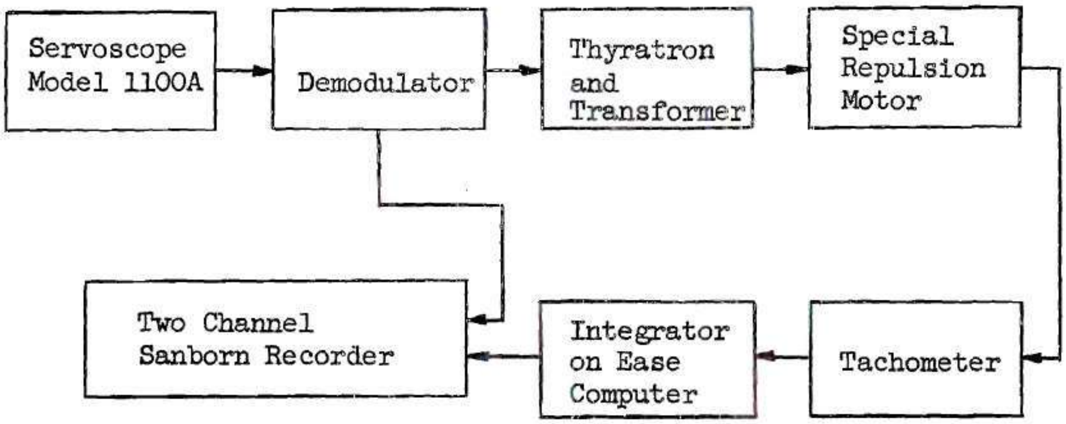



Figure 18. Equipment and Arrangements for Performing the Open-Loop Frequency Response Test

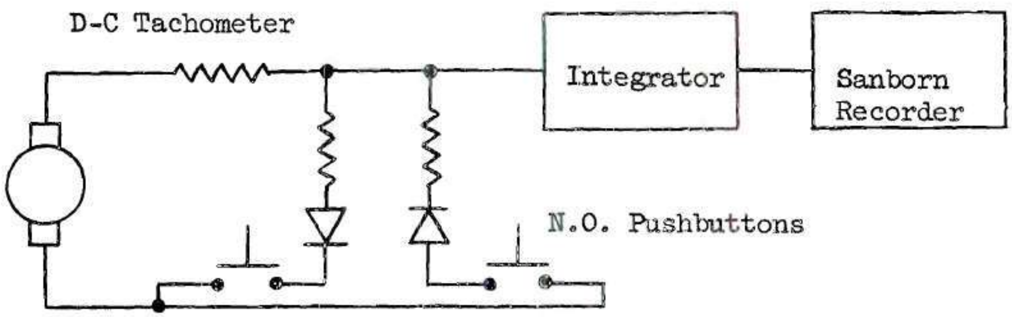



Figure 19. Electrical Zeroing Arrangement for the Open-Loop Frequency Response Test

open-loop system, the output of the tachometer, i.e. derivative of the output position, was used in order to evaluate the output position. But to avoid difficulties in evaluating the derivative of the output position caused by non-sinusoidal wave-form and variation in output voltage with frequency, the tachometer output was integrated. An integrator on the Berkeley Ease Computer was used with one to one time scale. Integrating the derivative again brought in the difficulty of zero position drift of the open-loop system. However with the integrator, this zero position drift could easily be corrected electrically by using the circuit shown in Figure 19. With the system operating, the zero position drift was compensated by depressing one of the push-buttons depending on the direction of drift, thus rezeroing the integrator. The fact that the actual motor position, about which the system is sinusoidally operating has changed, is obviously immaterial.

The results of the open-loop frequency response test are given in Table 1 in Run No. 1 through 18. Run No. 19 was determined in a slightly different manner. This data was determined by using the closed-loop system as shown in Figure 2 with the exception that the derivative feedback loop, i.e. the d-c tachometer, was disconnected. For a low gain adjustment of the a-c amplifier, the system was stable.

The gain of the a-c amplifier was then increased until the system became unstable and the system oscillated. By experimentation, a value of gain for the a-c amplifier could be found which was just sufficient to maintain the system in the oscillating or unsteady state. At this gain value the oscillating frequency was determined. The data are given in Run No. 19 of Table 1. This gain setting also corresponds with the gain setting used in all the dynamic tests. While at this value of gain, the system is unstable with only position feedback, the derivative feedback provides the necessary compensation to stabilize the system.

Because of the large magnitude of the output displacements at angles close to 180 degrees, a Nyquist plot of the data would not indicate clearly the shape of the open-loop curve. For the Nyquist plot where the $-1 + j0$ point has a normal relative scale on the graph, only a small portion of the data could be plotted and this would appear as a line lying very near the 180 degree axis.

However from an examination of the data in Table 1, the shape of the open-loop curve can be estimated. The polar plot of the open-loop curve is of the general shape which might be expected of this type system. The magnitude of the output approaches infinity along the $-j\omega$ axis as zero frequency is approached. The magnitude of the output approaches zero along the $+j\omega$ axis as the frequency is increased toward infinity.

A diagram of the test set-up used to evaluate the closed-loop response of the system is shown in Figure 20. The test arrangement is basically one that is suggested by the manufacturer of the Servoscope (8). A slight modification of the basic test set-up was required in order to provide the required push-pull input to the a-c amplifier.

A brief explanation of the operation of the circuit is given to explain some of the problems involved in using the circuit. The output

Table 1. Data of the Open-Loop Frequency Response Test

Input displacement - .02 radians

| Run | Input Frequency (cps) | Output Displacement (radians) | Output Phase (angle degrees) |
|---|---|---|---|
| 1 | 0.12 | 30.2 | 121 |
| 2 | 0.18 | 13.7 | 132 |
| 3 | 0.23 | 9.32 | 135 |
| 4 | 0.29 | 6.83 | 140 |
| 5 | 0.34 | 4.96 | 141 |
| 6 | 0.41 | 4.22 | 148 |
| 7 | 0.48 | 2.73 | 151 |
| 8 | 0.55 | 2.11 | 157 |
| 9 | 0.64 | 1.61 | 163 |
| 10 | 0.73 | 1.3 | 165 |
| 11 | 0.82 | 1.1 | 166 |
| 12 | 0.96 | 0.87 | 166 |
| 13 | 1.09 | 0.68 | 167 |
| 14 | 1.16 | 0.46 | 168 |
| 15 | 1.31 | 0.37 | --- |
| 16 | 1.47 | 0.31 | --- |
| 17 | 1.61 | 0.28 | 170 |
| 18 | 1.85 | 0.23 | --- |
| 19 | 5.5 | 0.02 | 180 |

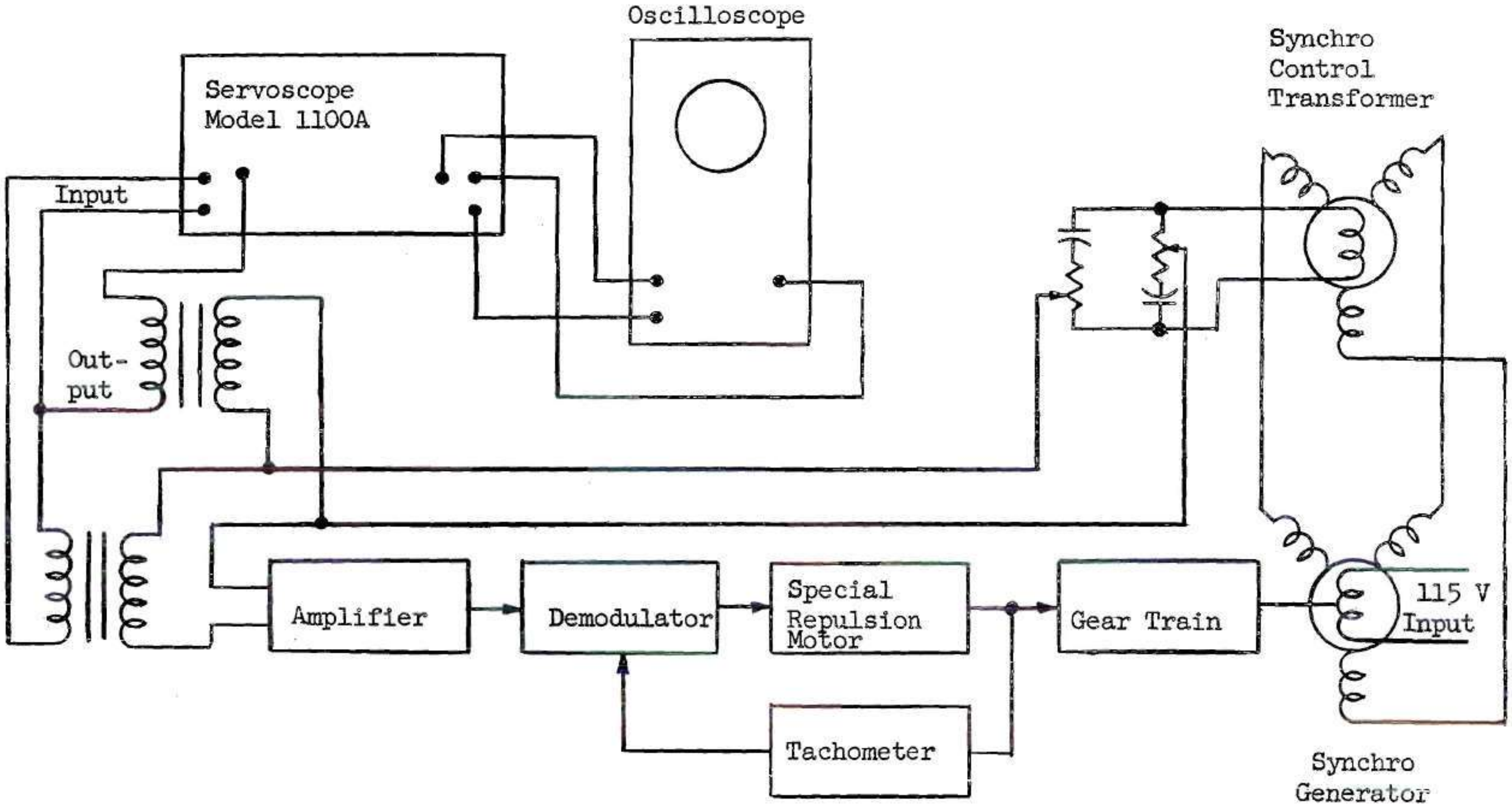



Figure 20. Equipment and Arrangements for Performing the Closed-Loop Frequency Response Test

signal of the Servoscope which is a suppressed carried modulated signal is applied to the primary of transformer T-1. The secondary of the transformer T-1 is in series with the output of the synchro control transformer. The resultant signal is applied to the grids of the push-pull input of the a-c amplifier. Transformer T-2 is connected across the synchro control transformer output so that the relative magnitude and phase of this output may be displayed on the Y axis of the oscilloscope. Because of the inherent phase shift of approximately 10 degrees in the carrier frequency of the synchros, a phase shift network is required across the output of the synchro control transformer. This is adjusted until the 60-cycle carrier of the Servoscope output signal and the 60-cycle carrier of the synchro control transformer output are in phase. To the X axis of the oscilloscope is applied a reference suppressed carrier modulated signal or a triggered sweep. Either of these two signals form a reference phase from which the phase shift of the output of the control system can be determined. The phase of the output signal of the Servoscope may be shifted at any angle from zero to 360 degrees with respect to this reference phase. The output signal of the Servoscope is the input signal to the control system. Consequently, by shifting the phase of the input signal to the control system, the phase of the output of the control system may be shifted until it is in phase with the reference phase of the Servoscope. After this is done, the phase difference between the output and input of the control system may be determined by reading the dial of the Servoscope of the relative phase difference between the output of the Servoscope and that of the reference phase.

Initially the a-c amplifier had a 2500 ohms potentiometer from each leg of the input to ground. This caused unsatisfactory operation of the test set-up. This problem was corrected by replacing the 2500 ohm potentiometers with approximately 1/2 megohm voltage dividers. The resistors in the voltage dividers were chosen so that the gain of the a-c amplifier for a given input signal was the same as before.

The results of the closed-loop frequency response test are given in Table 11, Appendix E. These results are clearly shown graphically in Figure 21. The response of the system is essentially flat for frequencies up to one cps. As an accurate positioning system where a steady state position is maintained, it would be most useful in this range.

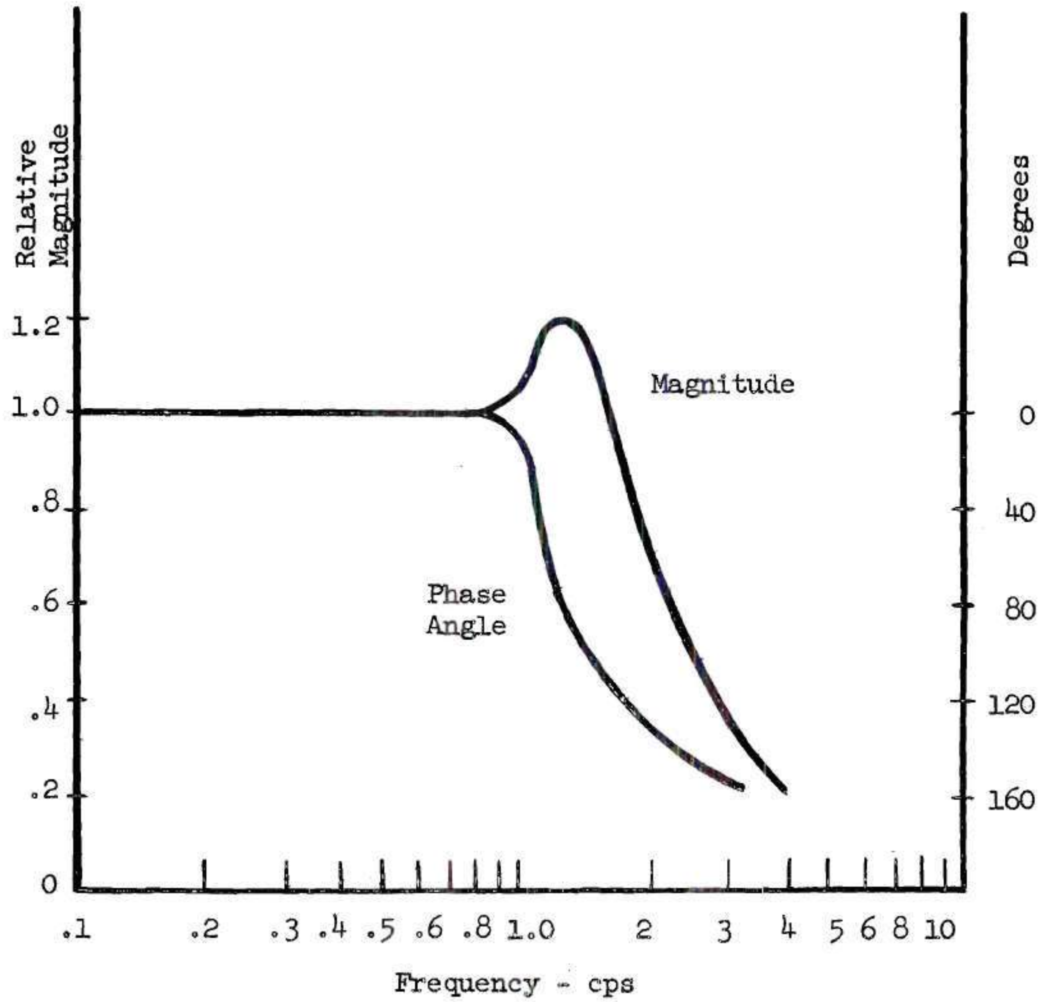



Figure 21. Magnitude and Phase Angle of the Closed-Loop Frequency Response for the Control System

# CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

Test results indicate that the system operates satisfactorily as a positioning servomechanism.

Performance of the Special Repulsion Motor.--The special repulsion motor used for the powering device of the positioning system was constructed and tested. Fifty-five volts proved to be the optimum input voltage to the stator winding. At this voltage the best efficiency of 46 to 47 per cent was obtained. The power output was 324 and 370 watts respectively. Fifty volts appeared to be the optimum choice due to internal heating of the motor above this voltage. At input voltages of 60 volts and above, saturation of the iron in the stator occurred. The stalled torque was 4.4 lb-ft with an input of 50 volts to the stator winding.

A double layer, lap, full-pitch winding was used on the stator. Two sets of taps spaced 20 electrical degrees each side of the electrical neutral on the stator winding were used in order to provide the reversing action required of the motor. (The brush axis concides with the electrical neutral.) Applying the input voltage to one set of taps produced rotation of the motor in one direction and applying the input voltage to the other set of taps reversed the direction of rotation. This method of reversing the motor was very satisfactory. Due to mechanical construction of the motor, the taps on the stator were located 20 electrical degrees with respect to the electrical neutral. From tests on the motor, 18 electrical degrees was found to be the setting for maximum stalled torque output. Therefore for all practical purposes the 20 electrical degree setting was the optimum setting. As the special repulsion motor is a four-pole motor, in order to provide the reversing action there must be two sets of taps for each pole pair. However the electrical balance of the stator winding was good and similar taps for each pole pair could be connected together without adverse effect. Therefore only two sets of taps for the input voltage were required thus simplifying the motor control circuit.

Performance of the System.--The control system utilizing the special repulsion motor operated satisfactorily. For all practical purposes, the position error was zero. The dead zone of the system was negligibly small. The velocity error was also small. As shown by the plot in Figure 17, the maximum velocity error is approximately 3 degrees. This occurs with an input to the synchro control transformer of about 46 rpm. This is about the maximum input that the control system can respond to without saturating. Difficulties with the gear train that connects the motor to the synchro generator resulted in some instability of the system during the velocity test. Difficulties with the gear train also made it impossible to obtain satisfactory transient response data for a step position input to the control system.

The results of the open-loop frequency response test as given in Table 1 show that the system is underdamped. The frequency for 180 degrees phase shift occurs at 5.5 cps.

As shown by the plot in Figure 21 of the closed-loop response of the system, the response of the system is essentially flat for frequencies up to one cps. Thus as an accurate positioning system where a steady state position is maintained, it would be most useful in this range.

Recommendations.--The use of a repulsion motor of this construction in a positioning servomechanism possesses considerable merit. As it is desirable to simplify the system as much as possible, the circuit of Figure 8 is recommended in preference to the one actually used in the testing.

The motor and transformer construction should be such, as to allow the system to operate directly from the a-c lines. For the motor used in this study, 40 turns of #21 wire per coil would be approximately what is required to allow the motor to operate directly on 115 volts. The coil using the smaller wire will also be much easier to form and the dimensions should not be as critical.

For the motor control circuit a magnetic amplifier or saturable reactor might be used in place of the transformers and thyratrons. The frequency response in this case would be only slightly less than when the transformers and thyratrons are used.

# APPENDIX A

## DETERMINATION OF THE INERTIA AND FRICTION TORQUE OF THE CONTROL SYSTEM

Procedure.--Tests were performed in order to determine the inertia and friction torque of the control system. Components of the control system used in this test were the special repulsion motor, the d-c tachometer, the gear train and the synchro generator.

The tests were performed as follows: A voltage was applied to the motor. When the speed of the motor reached a steady state rpm, the applied voltage was removed and the motor was allowed to coast to a stop. By monitoring the motor speed from the maximum value to zero rpm, a curve of motor speed versus time was obtained, by recording the output of the d-c tachometer on the Sanborn Recorder.

Four sets of tests were performed. In set 1, the synchro generator and the gear train were connected to the motor. Set 2 is the same as set 1 except that a flywheel of known inertia (0.0052 slug-ft$^2$) was attached to the motor. In set 3, the motor alone was used. In set 4, the motor with the flywheel of known inertia was used. In all four sets, the d-c tachometer was attached to the motor but the inertia and friction torque of the tachometer are negligible.

Results.--At the maximum rpm, the motor inertia possesses a certain kinetic energy. When the applied voltage is removed, this kinetic energy is dissipated by the friction torques. These friction torques include the viscous friction torque and the coulomb friction torque. The viscous friction term is proportional to velocity while the coulomb friction term is a constant for all velocities. The curves obtained from the tests were straight lines. This indicated the viscous damping term was negligible.

The data obtained for the average of each of the four test sets with 50 volts initially applied to the stator is given in Table 2.

Table 2. Load Parameter Test Data

| Set | Maximum Motor Speed (rpm) | Time from Maximum to Zero (seconds) | Maximum Motor Speed (Radians/second) | Slope (Radians/second)$^2$ |
|---|---|---|---|---|
| 1 | 2465 | 2.10 | 258 | 123 |
| 2 | 2435 | 7.45 | 255 | 34 |
| 3 | 2470 | 2.05 | 259 | 127 |
| 4 | 2460 | 7.38 | 258 | 35 |

The values of the coulomb friction torque and the inertia of the system can be obtained by using the equation of the form

$$J\left(\frac{d\omega}{dt}\right) = F$$

where

$J$ - inertia

$\frac{d\omega}{dt}$ - slope of velocity curve

$F$ - friction torque.

Using the data of the tests of set 1 and 2

$$\left(\frac{d\omega}{dt}\right)_s = \frac{F}{J_s} \quad \text{and} \quad \left(\frac{d\omega}{dt}\right)_T = \frac{F}{J_s + J_F}$$

where:

$F$ = friction torque

$J_s$ = inertia of system

$\left(\frac{d\omega}{dt}\right)_s$ = slope of velocity curve of set 1

$\left(\frac{d\omega}{dt}\right)_T$ = slope of velocity curve of set 2

$J_F$ = inertia of the flywheel 0.0052 slug-ft$^2$

Examination of the test data of set 3 and 4 indicates that the inertia and the friction torque of the motor is approximately the same as that for the system. Therefore for all practical purposes, the load inertia and friction torque on the positioning system is that of the motor.

# APPENDIX B

## STARTING TORQUE DATA FOR THE SPECIAL REPULSION MOTOR

Table 3 of this appendix shows measurements of starting torque versus input voltage to the stator winding. This test was made with brush axis centered with respect to the two sets of taps on the stator. Therefore, the starting torque values given in this table are equally valid for either direction of rotation. Voltage, current and power measurements of the stator winding were recorded and the short circuit armature current measurements were recorded.

Table 3. Starting Torque Data for the Special Repulsion Motor

| Stator | | | | | | |
|---|---|---|---|---|---|---|
| Voltage (volts) | Current (amps) | Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor |
| 30 | 16.3 | 364 | 32.6 | 1.8 | 489 | 0.75 |
| 35 | 19.4 | 484 | 36.8 | 2.8 | 680 | 0.71 |
| 40 | 22.6 | 600 | 44.6 | 3.7 | 905 | 0.66 |
| 45 | 26.8 | 840 | 53 | 4.1 | 1205 | 0.70 |
| 50 | 30.5 | 1190 | 59 | 4.4 | 1525 | 0.78 |

# APPENDIX C

## PERFORMANCE DATA FOR THE SPECIAL REPULSION MOTOR

Tables 4 through 9 of this appendix show data of motor performance for various values of applied voltage across the stator winding. These tests were made with the brush axis centered with respect to the two sets of taps on the stator winding. Therefore motor performance in either direction of rotation are similar. The data for Tables 4 through 8 are for counter-clockwise rotation of the motor. The data in Table 9 are for clockwise rotation of the motor. The voltage, current, and power in the stator winding, the armature short circuit current, the output torque and rpm of the motor were recorded. From these data, the volt-amperes into the stator winding, the power factor, output power and efficiency of the motor were determined.

Table 4. Performance Data for the Special Repulsion Motor with 40 Volts Applied Across the Stator Winding Counter-Clockwise Rotation of Motor

| Speed (rpm) | Stator Current (amps) | Stator Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
|---|---|---|---|---|---|---|---|---|
| 2430 | 7.8 | 256 | 14.8 | 0.1 | 312 | 0.82 | 40 | 16 |
| 1760 | 9.7 | 336 | 19.5 | 0.5 | 388 | 0.87 | 125 | 37 |
| 1510 | 11.2 | 396 | 22.8 | 0.8 | 448 | 0.88 | 161 | 41 |
| 1310 | 12.2 | 436 | 25.2 | 1.0 | 488 | 0.89 | 186 | 43 |
| 1100 | 13.6 | 480 | 27.6 | 1.3 | 542 | 0.89 | 195 | 41 |
| 900 | 14.6 | 516 | 30.0 | 1.5 | 584 | 0.88 | 192 | 37 |
| 720 | 15.9 | 552 | 32.3 | 1.8 | 634 | 0.87 | 129 | 32 |
| 560 | 17.0 | 580 | 34.4 | 2.0 | 680 | 0.85 | 159 | 27 |
| 390 | 18.3 | 596 | 36.4 | 2.3 | 733 | 0.81 | 125 | 21 |

Table 5. Performance Data for the Special Repulsion Motor with 45 Volts Applied Across the Stator Winding Counter-Clockwise Rotation of Motor

| | Stator | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Speed (rpm) | Current (amps) | Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
| 2680 | 8.4 | 300 | 15.6 | 0.1 | 378 | 0.79 | 50 | 17 |
| 2080 | 10.4 | 396 | 20.5 | 0.5 | 468 | 0.85 | 148 | 37 |
| 1790 | 11.4 | 446 | 22.8 | 0.8 | 511 | 0.87 | 191 | 43 |
| 1595 | 12.4 | 488 | 25.3 | 1.0 | 558 | 0.88 | 227 | 46 |
| 1390 | 13.5 | 536 | 27.5 | 1.3 | 608 | 0.88 | 247 | 46 |
| 1200 | 14.5 | 576 | 29.6 | 1.5 | 653 | 0.88 | 256 | 44 |
| 1000 | 15.9 | 624 | 32.4 | 1.8 | 716 | 0.87 | 249 | 40 |
| 880 | 17.1 | 672 | 34.8 | 2.0 | 770 | 0.87 | 250 | 37 |
| 760 | 18.1 | 708 | 37.0 | 2.3 | 815 | 0.87 | 243 | 34 |
| 660 | 19.3 | 740 | 39.0 | 2.5 | 869 | 0.85 | 234 | 32 |
| 580 | 20.1 | 760 | 40.6 | 2.8 | 905 | 0.84 | 226 | 30 |
| 420 | 21.2 | 760 | 42.2 | 3.0 | 965 | 0.79 | 176 | 23 |

Table 6. Performance Data for the Special Repulsion Motor with 50 Volts Applied Across the Stator Winding Counter-Clockwise Rotation of Motor

| Speed (rpm) | Stator Current (amps) | Stator Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
|---|---|---|---|---|---|---|---|---|
| 2810 | 10.0 | 380 | 18.5 | 0.2 | 500 | 0.76 | 80 | 21 |
| 2330 | 11.5 | 420 | 22.5 | 0.5 | 575 | 0.81 | 166 | 35 |
| 2010 | 12.4 | 516 | 24.6 | 0.8 | 620 | 0.83 | 214 | 42 |
| 1820 | 13.4 | 596 | 26.8 | 1.0 | 670 | 0.89 | 259 | 43 |
| 1630 | 14.4 | 636 | 29.3 | 1.3 | 720 | 0.88 | 290 | 46 |
| 1520 | 15.6 | 690 | 31.8 | 1.5 | 780 | 0.88 | 324 | 47 |
| 1300 | 16.8 | 752 | 34.5 | 1.8 | 842 | 0.89 | 323 | 43 |
| 1200 | 17.6 | 784 | 36.2 | 2.0 | 880 | 0.89 | 341 | 44 |
| 1080 | 18.7 | 820 | 39.5 | 2.3 | 935 | 0.87 | 345 | 42 |
| 920 | 20.0 | 880 | 40.4 | 2.5 | 1000 | 0.88 | 327 | 37 |
| 800 | 20.7 | 904 | 41.6 | 2.8 | 1033 | 0.87 | 312 | 35 |
| 700 | 21.8 | 930 | 43.8 | 3.0 | 1090 | 0.85 | 298 | 32 |
| 610 | 22.7 | 940 | 44.6 | 3.3 | 1135 | 0.83 | 282 | 30 |

Table 7. Performance Data for the Special Repulsion Motor with 55 Volts Applied Across the Stator Winding Counter-Clockwise Rotation of Motor

| | Stator | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Speed (rpm) | Current (amps) | Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
| 3030 | 11.6 | 430 | 19.5 | 0.1 | 638 | 0.67 | 43 | 10 |
| 2400 | 12.3 | 536 | 23.5 | 0.5 | 677 | 0.79 | 171 | 32 |
| 2180 | 13.6 | 610 | 26.0 | 0.8 | 748 | 0.82 | 232 | 38 |
| 1980 | 14.5 | 664 | 28.3 | 1.0 | 798 | 0.83 | 281 | 46 |
| 1810 | 14.8 | 688 | 29.5 | 1.3 | 813 | 0.85 | 321 | 47 |
| 1610 | 15.7 | 740 | 31.4 | 1.5 | 863 | 0.86 | 343 | 46 |
| 1490 | 16.9 | 810 | 34.0 | 1.8 | 930 | 0.87 | 370 | 46 |

Table 8. Performance Data for the Special Repulsion Motor with 60 Volts Applied Across the Stator Winding Counter-Clockwise Rotation of Motor

| | Stator | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Speed (rpm) | Current (amps) | Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
| 3260 | 12.9 | 530 | 21.3 | 0.1 | 774 | 0.69 | 46 | 9 |
| 2680 | 13.6 | 614 | 24.4 | 0.5 | 816 | 0.75 | 190 | 31 |
| 2400 | 14.7 | 680 | 27.0 | 0.8 | 882 | 0.77 | 255 | 38 |
| 2220 | 15.7 | 756 | 29.4 | 1.0 | 943 | 0.80 | 315 | 42 |
| 2005 | 16.7 | 836 | 32.2 | 1.3 | 1000 | 0.84 | 355 | 42 |
| 1840 | 17.5 | 896 | 34.4 | 1.5 | 1050 | 0.85 | 392 | 44 |
| 1690 | 18.7 | 970 | 37.3 | 1.8 | 1120 | 0.87 | 420 | 43 |

Table 9. Performance Data for the Special Repulsion Motor with 45 Volts Applied Across the Stator Winding Clockwise Rotation of Motor

| Speed (rpm) | Stator Current (amps) | Stator Power (watts) | Armature Current (amps) | Output Torque (lb-ft) | Stator (volt-amperes) | Power Factor | Output Power (watts) | Efficiency (%) |
|---|---|---|---|---|---|---|---|---|
| 2680 | 8.3 | 296 | 15.6 | 0.1 | 374 | 0.79 | 50 | 17 |
| 2010 | 10.0 | 372 | 19.6 | 0.5 | 450 | 0.83 | 149 | 40 |
| 1740 | 11.0 | 430 | 21.7 | 0.8 | 495 | 0.87 | 185 | 43 |
| 1500 | 12.2 | 476 | 24.4 | 1.0 | 549 | 0.87 | 213 | 45 |
| 1320 | 13.4 | 516 | 26.5 | 1.3 | 602 | 0.86 | 234 | 45 |
| 1200 | 14.4 | 564 | 28.6 | 1.5 | 648 | 0.87 | 256 | 45 |
| 1030 | 15.4 | 596 | 30.6 | 1.8 | 692 | 0.86 | 256 | 43 |
| 890 | 16.4 | 640 | 32.8 | 2.0 | 760 | 0.86 | 253 | 40 |

# APPENDIX D

## CLOSED-LOOP VELOCITY TEST

Table 10 of this appendix shows data of the velocity test. In this test both the position feedback and the velocity feedback loops were closed and a velocity input signal was introduced. The angular difference between the synchro generator and synchro control transformer shafts was measured for various values of input velocity signals. The test was made for one direction of rotation of the control system.

Table 10. Closed-Loop Velocity Test for the Control System

| Velocity of Input Signal (rpm) | Velocity of Motor Output (rpm) | Position Error of Transformer-Generator (degrees) |
|---|---|---|
| 7.13 | 308 | 0.37 |
| 7.71 | 333 | 0.42 |
| 12.45 | 539 | 0.58 |
| 16.0 | 694 | 0.66 |
| 23.1 | 1000 | 0.85 |
| 25.7 | 1108 | 0.95 |
| 33.2 | 1438 | 1.49 |
| 42.8 | 1850 | 2.2 |
| 47.6 | 2055 | 3.7 |
| 49.9 | 2155 | 6.4 |

# APPENDIX E

## CLOSED-LOOP FREQUENCY RESPONSE TEST

Table 11 of this appendix shows the data of the closed-loop frequency response test. In this test both the position feedback and the velocity feedback loops were closed and a sinusoidal input signal was introduced. The output amplitude and phase angle with respect to the input were measured. The frequency of the input signal was varied over a range of .15 to 4 cps.

Table 11. Closed-Loop Frequency Response Test for the Control System

| Run No. | Frequency of Input Signal (cps) | Relative Amplitude Output/Input | Phase Angle Difference (degrees) |
|---|---|---|---|
| 1 | 0.15 | 1.00 | 0 |
| 2 | 0.3 | 1.00 | 0 |
| 3 | 0.4 | 1.00 | 0 |
| 4 | 0.5 | 1.00 | 0 |
| 5 | 0.6 | 1.00 | 0 |
| 6 | 0.8 | 1.04 | 2 |
| 7 | 1.0 | 1.06 | 10 |
| 8 | 1.2 | 1.18 | 72 |
| 9 | 1.4 | 1.16 | 101 |
| 10 | 1.6 | 0.94 | 113 |
| 11 | 1.8 | 0.78 | 124 |
| 12 | 2.0 | 0.62 | 135 |
| 13 | 2.5 | 0.56 | 145 |
| 14 | 3.0 | 0.39 | 154 |
| 15 | 3.5 | 0.27 | 160 |
| 16 | 4.0 | 0.21 | 165 |

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